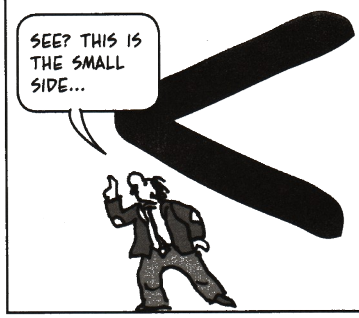
LET'S DO A LITTLE WARM-UP EXERCISE WITH THIS NEW LANGUAGE BY DESCRIBING SOME **NEW SYMBOLS** IN TERMS OF VARIABLES. HERE THEY ARE NOW...

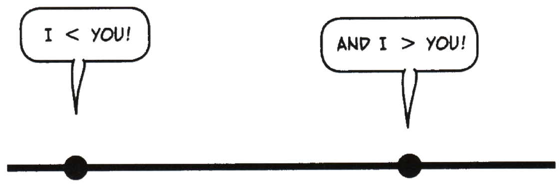
THE SYMBOLS ARE RELATIVES OF THE FAMILIAR EQUALS SIGN =. < MEANS "IS LESS THAN," AND > MEANS "IS GREATER THAN."



IN TERMS OF VARIABLES, WE'D PUT IT THIS WAY: SUPPOSE a AND b ARE **ANY TWO NUMBERS.**

$a < b$ MEANS THAT a IS TO THE LEFT OF b ON THE NUMBER LINE.

$a > b$ MEANS THAT a IS TO THE RIGHT OF b ON THE NUMBER LINE.

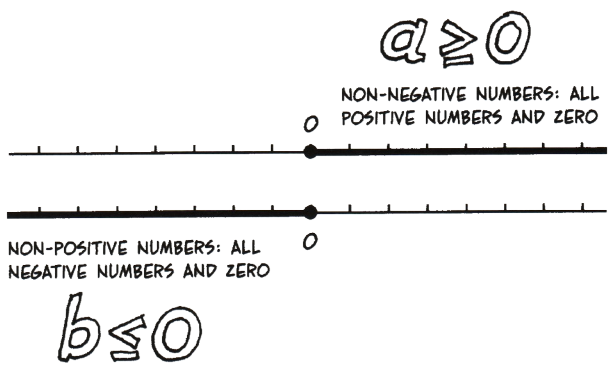


$a > 0$ SAYS THAT a IS POSITIVE, WHILE $a < 0$ SAYS THAT a IS NEGATIVE.

WE ALSO SOMETIMES USE THE SYMBOLS ≤, "IS LESS THAN OR EQUAL TO," AND ≥, "IS GREATER THAN OR EQUAL TO." SO

$a \geq 0$

MEANS THAT a COULD BE ANY POSITIVE NUMBER, OR POSSIBLY ZERO. a, WE WOULD SAY, IS **NON-NEGATIVE.** THE NON-POSITIVE NUMBERS WOULD BE THOSE NUMBERS b WITH $b \leq 0$.



LAWS OF COMBINATION

WHEN COMBINING NUMBERS OR VARIABLES, WE MUST ALWAYS FOLLOW THE LAW. OTHERWISE, WE MIGHT BE GUILTY OF GETTING THE WRONG ANSWER, AND THEN WHO KNOWS WHAT?

THE FIRST LAWS SAY THAT IN **SOME** EXPRESSIONS THE ORDER OF **NUMBERS** DOESN'T MATTER:

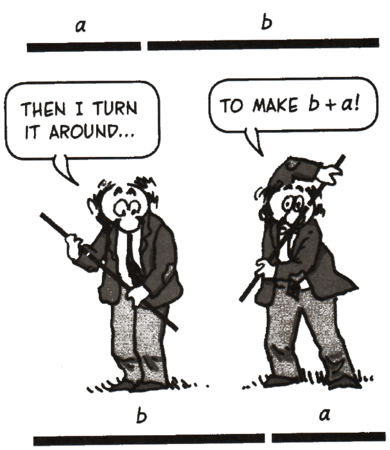
COMMUTATIVE LAWS:

IF a AND b ARE ANY TWO NUMBERS, THEN

$$a+b=b+a$$
$$ab=ba$$

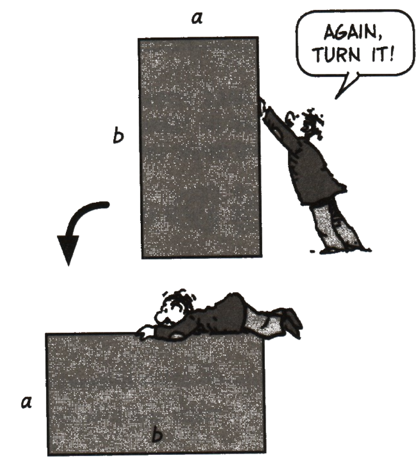
(THIS IS REALLY TWO LAWS, ONE FOR SUMS AND ONE FOR PRODUCTS.)

HERE'S THE PICTURE FOR ADDITION (DRAWN FOR POSITIVE NUMBERS ONLY). $a+b$ IS THE LENGTH OF THIS STICK...



TURNING SOMETHING AROUND DOESN'T CHANGE ITS LENGTH, SO $a+b=b+a$.

THE PRODUCT ab IS THE AREA OF A RECTANGLE OF LENGTH a AND HEIGHT b.



THE TIPPED-OVER RECTANGLE HAS AREA ba. TURNING SOMETHING DOESN'T CHANGE ITS AREA, SO $ba = ab$.

Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.

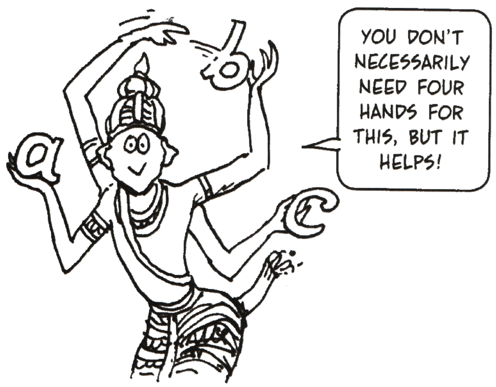
SOMETIMES THE ORDER OF **OPERATIONS** DOESN'T MATTER.

ASSOCIATIVE LAWS:

IF a, b, AND c ARE ANY THREE NUMBERS, THEN

$$(a+b)+c = a+(b+c)$$
$$(ab)c = a(bc)$$

WHEN ADDING OR MULTIPLYING **ONLY**, GROUPING ("ASSOCIATION") DOESN'T MATTER.



Associative Examples:

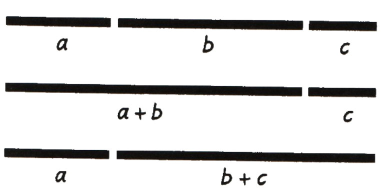
1. $(2+3)+4 = 5+4 = 9$

$2+(3+4) = 2+7 = 9$

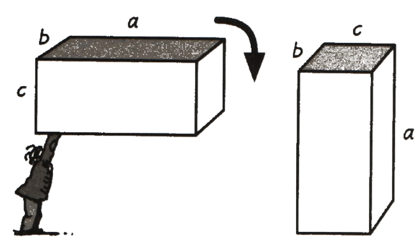
2. $(5\times 3)\times 6 = 15\times 6 = 90$

$5\times(3\times 6) = 5\times 18 = 90$

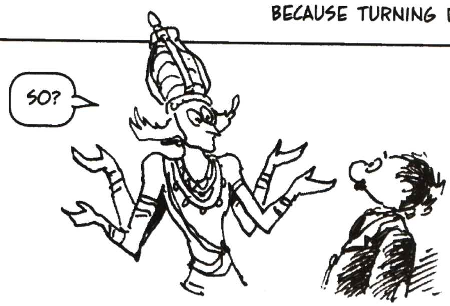
THE PICTURE FOR ADDITION (OF POSITIVE NUMBERS) IS SUPER-SIMPLE. ALL THESE LINES OBVIOUSLY HAVE THE SAME TOTAL LENGTH. IT DOESN'T MATTER WHERE YOU BREAK THEM.



AND FOR MULTIPLICATION...



ONE BLOCK HAS VOLUME $(ab)c$. THE OTHER HAS VOLUME $a(bc)$. THESE MUST BE EQUAL, BECAUSE TURNING DOESN'T AFFECT VOLUME.



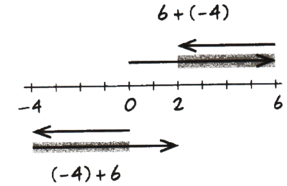
Minus Signs and the Laws

WE PICTURED THE COMMUTATIVE LAW $a+b=b+a$ WITH TWO POSITIVE NUMBERS, BUT THE LAW IS JUST AS TRUE WHEN a, b, OR BOTH ARE NEGATIVE. THAT'S BECAUSE ADDITION IS **DEFINED** TO BE THE SAME IN EITHER ORDER. (SEE P. 19 OR P. 47.) FOR INSTANCE,

$$\left.\begin{array}{l} 6+(-4) \\ (-4)+6 \end{array}\right\}$$

SUBTRACT 4 FROM 6, THEN GIVE THE ANSWER THE SAME SIGN AS 6 BECAUSE $|6| > |-4|$.

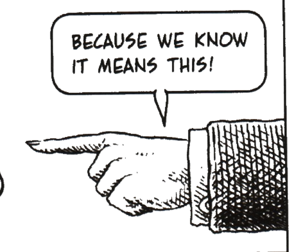
THE ASSOCIATIVE LAW ALSO APPLIES TO NEGATIVE NUMBERS.



SEEN WITH ARROWS, $6+(-4)$ TAKES 4 FROM THE **HEAD** END OF THE 6-ARROW, WHILE $-4+6$ TAKES 4 FROM THE **TAIL** END OF THE 6-ARROW. THE RESULT IS THE SAME: 2.

THIS MAKES IT OKAY TO LEAVE OUT PARENTHESES IN "SUMS" THAT INCLUDE BOTH PLUS AND MINUS SIGNS, LIKE THIS ONE:

$$2-4-5+3$$
$$= 2+(-4)+(-5)+3$$



AND WE CAN WRITE THE EXPRESSION IN ANY ORDER, AS LONG AS THE MINUS SIGNS STICK TO THEIR NUMBERS. THESE ARE ALL THE SAME:

$$-4-5+3+2$$
$$-4+3-5+2$$
$$3+2-5-4$$
$$-5+2-4+3$$
$$2-4+3-5$$

ETC.!

Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.