

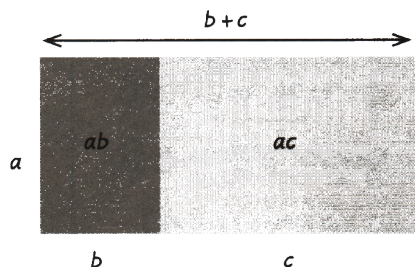
SO FAR, OUR LAWS HAVE ALLOWED SHUFFLING AND REGROUPING WITHIN SUMS AND PRODUCTS. OUR NEXT (AND FINAL) LAW OF COMBINATION IS DIFFERENT. IT DESCRIBES WHAT HAPPENS WHEN **TIMES** MEETS **PLUS**.

DISTRIBUTIVE LAW:

IF a , b AND c ARE ANY NUMBERS, THEN

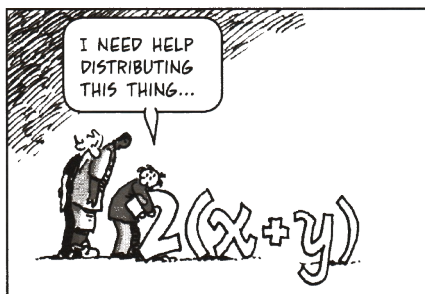
$$a(b+c) = ab+ac$$

THE PRODUCT OF A NUMBER TIMES A SUM IS THE SUM OF THE TWO "PARTIAL PRODUCTS." MULTIPLICATION "DISTRIBUTES" OVER ADDITION (AGAIN, IT DOESN'T MATTER WHETHER THE NUMBERS a , b , AND c ARE POSITIVE, NEGATIVE, OR ZERO).



THE LARGE RECTANGLE, WITH AREA $a(b+c)$, IS MADE UP OF TWO SMALLER RECTANGLES WITH AREAS ab AND ac .

WE USE THE ASSOCIATIVE AND COMMUTATIVE LAWS ALMOST MINDLESSLY. OF COURSE $2+3 = 3+2$!! THE DISTRIBUTIVE LAW, ON THE OTHER HAND, CALLS FOR SOME CARE, BECAUSE WE'RE PUSHING A FACTOR ONTO MORE THAN ONE ITEM INSIDE THE PARENTHESES.

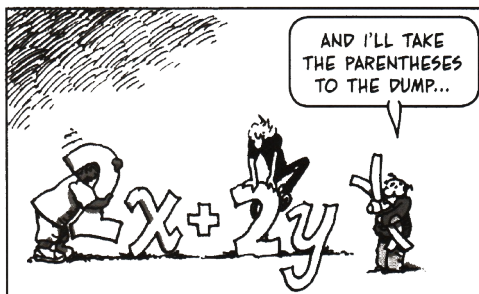


Numerical Example:

$$2(5+7) = 2(12) = 24$$

$$\text{and also } = 2 \times 5 + 2 \times 7 \\ = 10 + 14 \\ = 24$$

CHECK!



Examples with Variables:

$$1. 3(x+1) = 3x + (3)(1) = 3x + 3$$

$$2. 2a(x+3) = 2ax + 6a$$

NOTE THAT ORDER DOESN'T MATTER:

$$3. P + \frac{1}{2}P = (1 + \frac{1}{2})P = \frac{3}{2}P$$

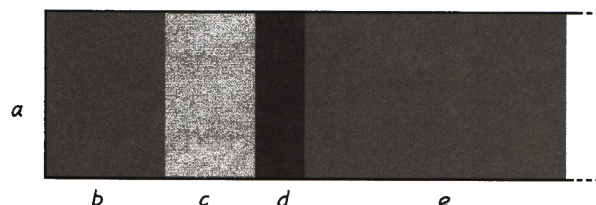
$$4. ax + 2x = (a+2)x$$



A FEW THINGS TO NOTICE ABOUT THE DISTRIBUTIVE LAW:

Multiplication distributes over long sums.

$$a(b+c+d+e+\dots) = ab+ac+ad+ae+\dots$$

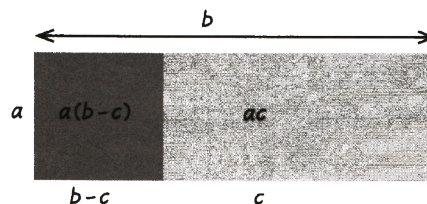


Multiplication distributes over subtraction:

$$a(b-c) = ab-ac$$

THIS IS TRUE BECAUSE SUBTRACTION IS "NEGATIVE ADDITION."

$$\begin{aligned} a(b-c) &= a(b+(-c)) && \text{DEFINITION OF SUBTRACTION} \\ &= ab + a(-c) && a \text{ DISTRIBUTES OVER ADDITION} \\ &= ab + a((-1)c) && -c = (-1)c \\ &= ab + (-1)(ac) && \text{SHUFFLING AND REGROUPING!} \\ &= ab + (-ac) && (-1)ac = -ac \\ &= ab - ac && \text{DEFINITION OF SUBTRACTION} \end{aligned}$$



Negation distributes!

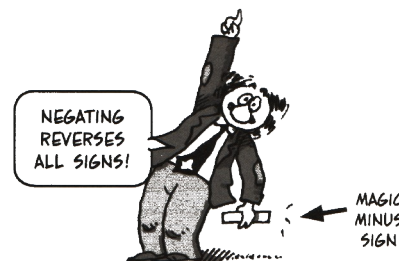
$$-(a+b) = -a-b$$

THIS IS TRUE BECAUSE NEGATION IS THE SAME AS MULTIPLICATION BY -1 .

$$\begin{aligned} -(a+b) &= (-1)(a+b) \\ &= (-1)a + (-1)b \\ &= -a-b \end{aligned}$$

DISTRIBUTING A MINUS SIGN OVER A SUBTRACTION LOOKS LIKE THIS:

$$\begin{aligned} -(a-b) &= -a+b \\ &= b-a \end{aligned}$$



SOMETIMES WE WANT TO "UNDISTRIBUTE," FOR EXAMPLE WHEN ADDING MULTIPLES OF A SINGLE VARIABLE SUCH AS $3x + 2x$.

$$3x + 2x = (3+2)x = 5x$$



MULTIPLES OF A VARIABLE ADD AS EXPECTED. DIDN'T WE PROMISE NO AWFUL SURPRISES?

3 APPLES PLUS 2 APPLES MAKE 5 APPLES, AND THE SAME FOR x !

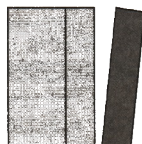


Reminder: THIS WORKS WITH ANY MULTIPLES, NOT JUST POSITIVE WHOLE NUMBERS. FOR INSTANCE,

$$2y - \frac{y}{2} = (2 - \frac{1}{2})y = \frac{3}{2}y \text{ OR } \frac{3y}{2}$$



$2y$



TAKE AWAY $y/2$



LEAVES $(1\frac{1}{2})y$ OR $\frac{3y}{2}$

also:

$$P + .3P = (1 + .3)P = (1.3)P$$

(BECAUSE $P = 1 \cdot P$ WITH AN UNWRITTEN 1.)

$$6z - 2z = 4z$$

$$\frac{x}{2} + \frac{x}{3} = (\frac{1}{2} + \frac{1}{3})x = \frac{5}{6}x \text{ OR } \frac{5x}{6}$$

REAL-WORLD EXAMPLE: Discount Store

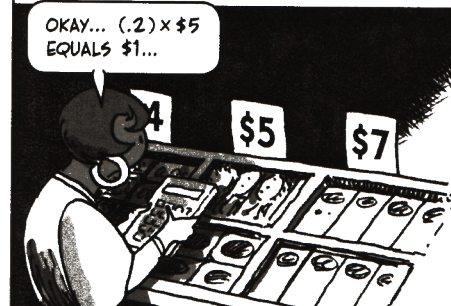
MY LOCAL BARGAIN STORE IS HAVING A 20%-OFF SALE. EVERYTHING IN THE STORE IS REDUCED BY AN AMOUNT EQUAL TO 20% OF THE MARKED PRICE.

I LOVE SALES, BUT WHAT DOES THIS HAVE TO DO WITH THE DISTRIBUTIVE LAW?



FINDING THE DISCOUNTED (SALE) PRICE OF AN ITEM WOULD SEEM TO TAKE TWO STEPS. STEP 1: FIND THE DISCOUNT BY MULTIPLYING THE PRICE BY .2 (THAT'S 20%, OR 20/100).

OKAY... $(.2) \times \$5$ EQUALS \$1...



NOW APPLY THE DISTRIBUTIVE LAW. WE KNOW THAT $P = 1 \cdot P$ (THE 1 BEING UNWRITTEN), SO

$$P - .2P = (1 - .2)P = .8P$$

YES! $.8 \times \$5$ EQUALS \$4 TOO!



EVEN BETTER: MAYBE WE WANT THE TOTAL COST OF SEVERAL ITEMS, SAY FOUR OF THEM WITH MARKED PRICES P , Q , R , AND S . THEIR SALES PRICES ADD UP TO $.8P + .8Q + .8R + .8S$, BUT—

$$.8P + .8Q + .8R + .8S = .8(P + Q + R + S)$$

I MAY NOT AGREE WITH EVERY LAW, BUT THIS IS A GOOD ONE!

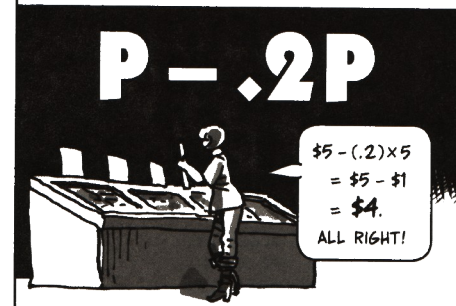


STEP 2: SUBTRACT THE DISCOUNT FROM THE MARKED PRICE. IN TERMS OF A VARIABLE, AN ITEM WITH A MARKED PRICE P HAS A DISCOUNT OF $.2P$ AND A SALE PRICE OF

$$P - .2P$$

$$\begin{aligned} \$5 - (.2) \times \$5 \\ = \$5 - \$1 \\ = \$4. \end{aligned}$$

ALL RIGHT!



IN REALITY, THEN, WE CAN FIND THE DISCOUNTED PRICE IN JUST ONE STEP: MULTIPLY THE MARKED PRICE BY $.8$!



IN OTHER WORDS, TO FIND THE TOTAL COST OF SEVERAL ITEMS, ADD ALL THEIR MARKED PRICES AND MULTIPLY BY $.8$. YOU NEVER HAVE TO KNOW THE DISCOUNT OF A SINGLE ITEM!!

WHAT'LL YOU DO WITH ALL THE TIME YOU SAVE CALCULATING?

SHOP!

