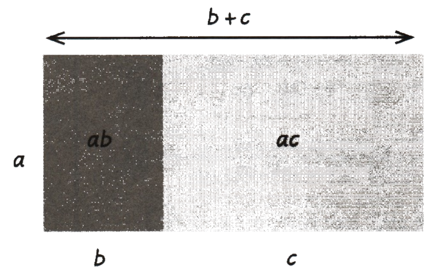
SO FAR, OUR LAWS HAVE ALLOWED SHUFFLING AND REGROUPING **WITHIN** SUMS AND PRODUCTS. OUR NEXT (AND FINAL) LAW OF COMBINATION IS DIFFERENT. IT DESCRIBES WHAT HAPPENS WHEN **TIMES** MEETS **PLUS**.

DISTRIBUTIVE LAW:

IF a, b AND c ARE ANY NUMBERS, THEN

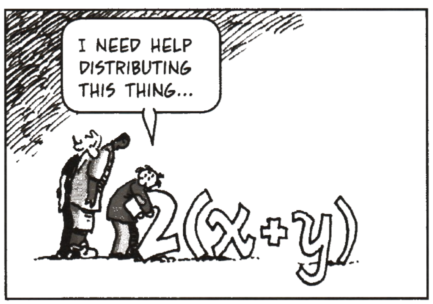
$$a(b+c) = ab + ac$$

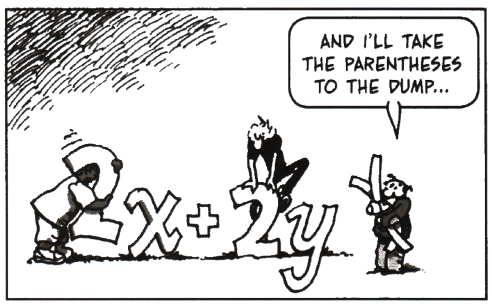
THE PRODUCT OF A NUMBER TIMES A SUM IS THE SUM OF THE TWO "PARTIAL PRODUCTS." MULTIPLICATION "DISTRIBUTES" OVER ADDITION (AGAIN, IT DOESN'T MATTER WHETHER THE NUMBERS a, b, AND c ARE POSITIVE, NEGATIVE, OR ZERO).



THE LARGE RECTANGLE, WITH AREA $a(b+c)$, IS MADE UP OF TWO SMALLER RECTANGLES WITH AREAS ab AND ac.

WE USE THE ASSOCIATIVE AND COMMUTATIVE LAWS ALMOST MINDLESSLY. OF **COURSE** $2+3 = 3+2$!! THE DISTRIBUTIVE LAW, ON THE OTHER HAND, CALLS FOR SOME CARE, BECAUSE WE'RE PUSHING A FACTOR ONTO MORE THAN ONE ITEM INSIDE THE PARENTHESES.





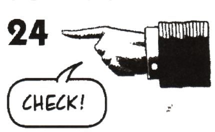
Numerical Example:

$$2(5+7) = 2(12) = 24$$

and also $= 2\times5 + 2\times7$

$= 10 + 14$

$= 24$



Examples with Variables:

1. $3(x+1) = 3x + (3)(1) = 3x + 3$
2. $2a(x+3) = 2ax + 6a$

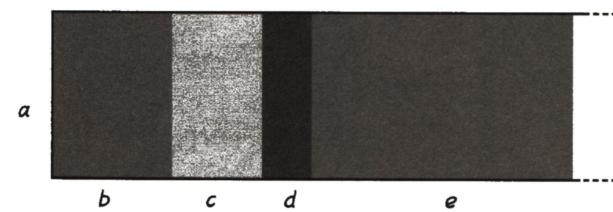
NOTE THAT ORDER DOESN'T MATTER:

3. $P + \frac{1}{2}P = (1 + \frac{1}{2})P = \frac{3}{2}P$
4. $ax + 2x = (a+2)x$

A FEW THINGS TO NOTICE ABOUT THE DISTRIBUTIVE LAW:

Multiplication distributes over long sums.

$$a(b+c+d+e+\ldots) = ab+ac+ad+ae+\ldots$$

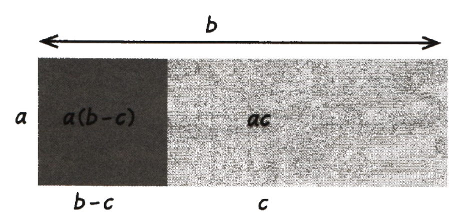


Multiplication distributes over subtraction:

$$a(b-c) = ab - ac$$

THIS IS TRUE BECAUSE SUBTRACTION IS "NEGATIVE ADDITION."

$a(b-c)$	$= a(b+(-c))$	DEFINITION OF SUBTRACTION
	$= ab + a(-c)$	a DISTRIBUTES OVER ADDITION
	$= ab + a((-1)c)$	$-c = (-1)c$
	$= ab + (-1)(ac)$	SHUFFLING AND REGROUPING!
	$= ab + (-ac)$	$(-1)ac = -ac$
	$= ab - ac$	DEFINITION OF SUBTRACTION



Negation distributes!

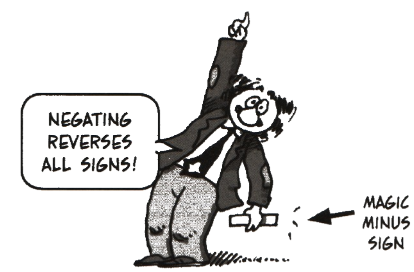
$$-(a+b) = -a-b$$

THIS IS TRUE BECAUSE NEGATION IS THE SAME AS MULTIPLICATION BY −1.

$$-(a+b) = (-1)(a+b)$$
$$= (-1)a + (-1)b$$
$$= -a - b$$

DISTRIBUTING A MINUS SIGN OVER A SUBTRACTION LOOKS LIKE THIS:

$$-(a-b) = -a+b$$
$$= b-a$$

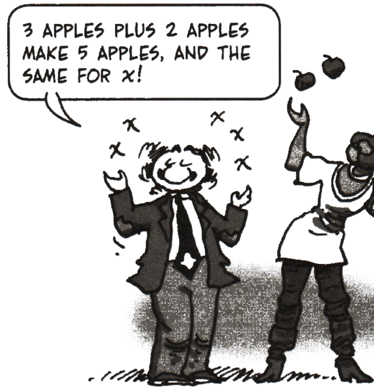


Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.

SOMETIMES WE WANT TO "UNDISTRIBUTE," FOR EXAMPLE WHEN ADDING MULTIPLES OF A SINGLE VARIABLE SUCH AS $3x + 2x$.

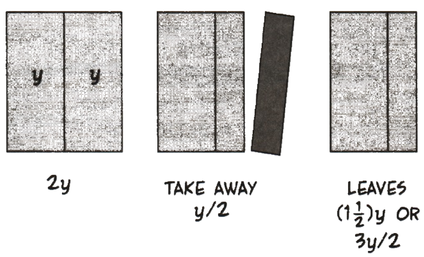
$$3x + 2x = (3+2)x$$
$$= 5x$$

MULTIPLES OF A VARIABLE ADD AS EXPECTED. DIDN'T WE PROMISE NO AWFUL SURPRISES?



Reminder: THIS WORKS WITH ANY MULTIPLES, NOT JUST POSITIVE WHOLE NUMBERS. FOR INSTANCE,

$$2y - \frac{y}{2} = \left(2 - \frac{1}{2}\right)y = \frac{3}{2}y \text{ OR } \frac{3y}{2}$$



also:

$$P + .3P = (1 + .3)P = (1.3)P$$

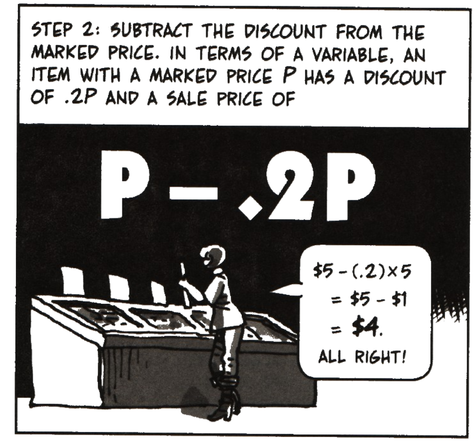
(BECAUSE $P = 1 \cdot P$ WITH AN UNWRITTEN 1.)

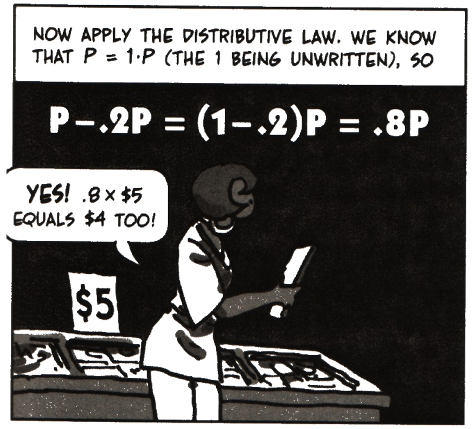
$$6z - 2z = 4z$$

$$\frac{x}{2} + \frac{x}{3} = \left(\frac{1}{2} + \frac{1}{3}\right)x = \frac{5}{6}x \text{ OR } \frac{5x}{6}$$

REAL-WORLD EXAMPLE: Discount Store

MY LOCAL BARGAIN STORE IS HAVING A 20%-OFF SALE. EVERYTHING IN THE STORE IS REDUCED BY AN AMOUNT EQUAL TO 20% OF THE MARKED PRICE.

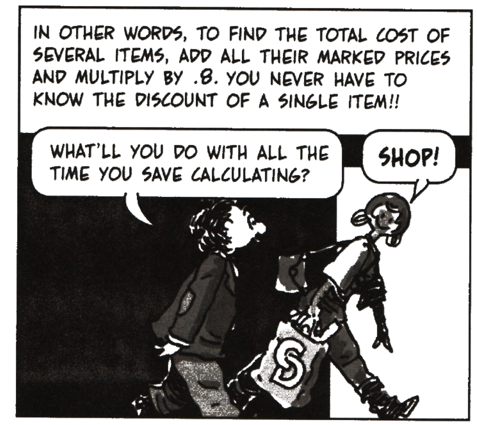




IN REALITY, THEN, WE CAN FIND THE DISCOUNTED PRICE IN JUST **ONE** STEP: **MULTIPLY THE MARKED PRICE** BY **.8!**

EVEN BETTER: MAYBE WE WANT THE TOTAL COST OF SEVERAL ITEMS, SAY FOUR OF THEM WITH MARKED PRICES P, Q, R, AND S. THEIR SALES PRICES ADD UP TO $.8P + .8Q + .8R + .8S$, BUT—

$$.8P + .8Q + .8R + .8S$$
$$= .8(P + Q + R + S)$$



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