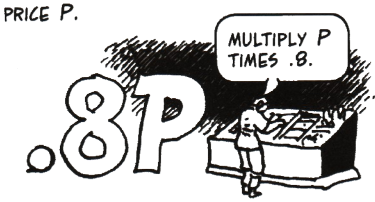
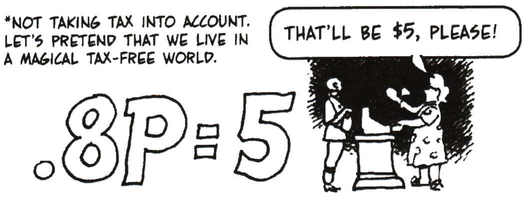
FOR INSTANCE, IN THE DISCOUNT STORE, THIS EXPRESSION DESCRIBES HOW TO CALCULATE THE SALE PRICE OF AN ITEM MARKED DOWN 20% FROM ITS ORIGINAL PRICE *P*.




WHEN THE CASHIER TELLS YOU WHAT YOU MUST ACTUALLY PAY, THAT'S AN EQUATION, A STATEMENT. IT SAYS THE SALE PRICE **IS EQUAL TO** SOME NUMBER.*

*NOT TAKING TAX INTO ACCOUNT. LET'S PRETEND THAT WE LIVE IN A MAGICAL TAX-FREE WORLD.





LIKE ANY STATEMENT OF FACT, AN EQUATION CAN BE **TRUE** OR **FALSE**.

$$2+2=3+1$$ TRUE

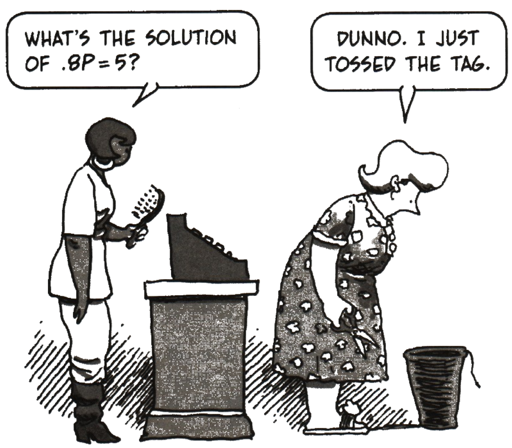
$$2+2=3$$ NOT SO TRUE!

WE USE THE SYMBOL $\neq$ TO MEAN "IS **NOT** EQUAL TO," AS IN

$$2+2\neq 3$$ TRUE

AN EQUATION CONTAINING A **VARIABLE** MAY BE TRUE FOR SOME VALUE OR VALUES OF THE VARIABLE AND NOT FOR OTHERS. THE EQUATION $2x+1=7$ IS TRUE WHEN $x=3$ BECAUSE $2(3)+1=7$, BUT FALSE WHEN $x=4$ BECAUSE $2(4)+1=9\neq 7$.

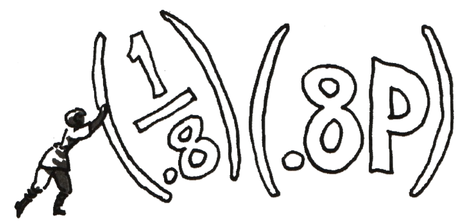
A VALUE OF THE VARIABLE THAT MAKES THE EQUATION TRUE IS CALLED A **SOLUTION** OF THE EQUATION. A SOLUTION IS SAID TO **SATISFY** OR **SOLVE** THE EQUATION. $x=3$ SATISFIES THE EQUATION $2x+1=7$. TRY SOME OTHER VALUES OF $x$. DO YOU FIND ANY OTHER SOLUTIONS?






SUPPOSE YOU BOUGHT SOMETHING MARKED DOWN 20% TO $5. **WHAT WAS THE ITEM'S ORIGINAL PRICE BEFORE THE MARKDOWN?** UNFORTUNATELY, THE CASHIER THREW AWAY THE PRICE TAG AND LEFT YOU WITH NOTHING BUT AN EQUATION TO PONDER:


THE EQUATION TELLS YOU THE VALUE OF 80% OF *P*, A FRACTION OF *P*. HOW WOULD YOU FIND THE VALUE OF *P* ITSELF, ALL OF *P*, THAT IS $1\cdot P$? HOW CAN YOU TURN $.8P$ INTO $1\cdot P$? ANSWER: **MULTIPLY BY THE RECIPROCAL OF .8** (OR DIVIDE BY .8, SAME THING).*




THAT WILL CLEAR AWAY THE FACTOR .8, BECAUSE

$$\frac{1}{.8}(.8P) = (\frac{.8}{.8})P = P$$

BUT THEN WHAT ABOUT THE **5** ON THE OTHER SIDE OF EQUATION?

WELL, IF BEING A TRUE EQUATION MEANS ANYTHING, IT MEANS THIS: $.8P$ AND 5 ARE REALLY THE **SAME NUMBER**. NATURALLY, THEN, **ANY MULTIPLE** OF $.8P$ AND 5 WILL ALSO BE EQUAL TO EACH OTHER. HOW COULD THEY NOT BE? SO...

$$\frac{1}{.8}(.8P)=\frac{1}{.8}(5)$$


NOW FOR THE ARITHMETIC:

$$\frac{1}{.8}(.8P) = (\frac{1}{.8})5$$

$$P = 5/.8 = \mathbf{6.25}$$

THE ORIGINAL PRICE WAS **$6.25.**

BECAUSE YOU'RE CONCERNED THAT YOUR REASONING MAY HAVE BEEN SHAKY, YOU CHECK TO MAKE SURE THAT $P=6.25$ REALLY DOES SATISFY THE EQUATION:

$$(.8)(6.25) \stackrel{?}{=} 5$$

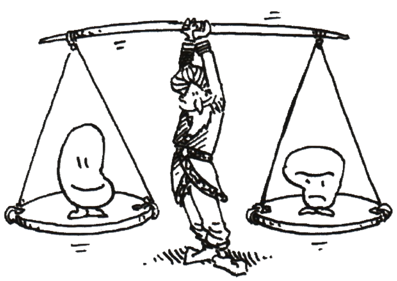
$$5=5$$


*IF YOU HATE THE DECIMAL, YOU CAN WRITE $.8 = 8/10 = 4/5$ AND ITS RECIPROCAL AS $5/4$.

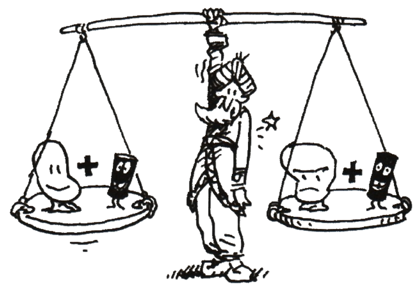
Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.

WE HAVE JUST MET ALGEBRA'S FIRST **BIG IDEA**: GIVEN ANY TRUE EQUATION, YOU CAN "DO THE SAME THINGS TO BOTH SIDES" AND THE RESULTING EQUATION WILL STILL BE TRUE. THIS IDEA COMES FROM THE INVENTOR OF ALGEBRA HIMSELF, MUHAMMAD OF KHWARIZM, OR **AL-KHWARIZMI** (780–850).

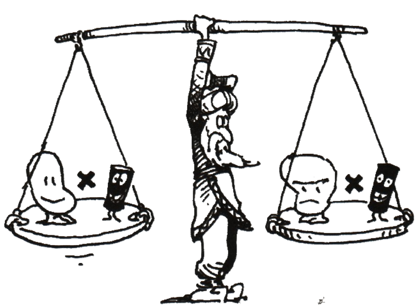

AL-KHWARIZMI THOUGHT OF AN EQUATION AS "BALANCED." THE EXPRESSIONS ON THE TWO SIDES, THOUGH THEY LOOK DIFFERENT, EXPRESS THE SAME NUMBER.



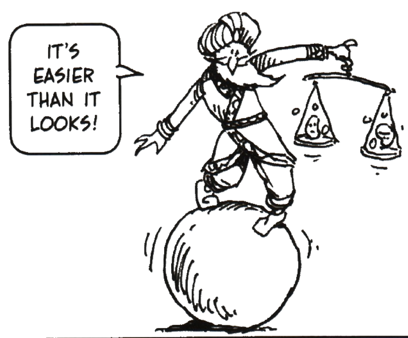
IF WE **ADD** THE SAME THING (NUMBER, EXPRESSION, WHATEVER) TO BOTH SIDES, THE SIDES STILL BALANCE—THEY'RE STILL EQUAL TO ONE ANOTHER.




WE CAN ALSO **MULTIPLY** BOTH SIDES BY THE SAME THING AND STAY IN BALANCE.

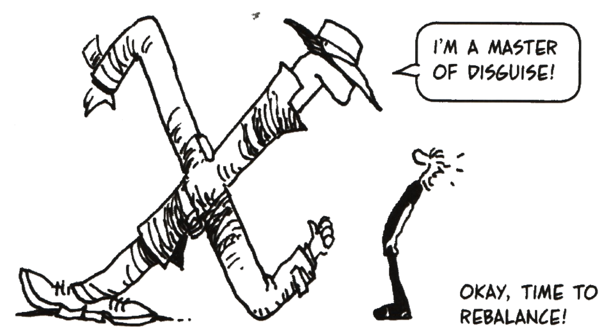



WE CAN SOLVE MANY EQUATIONS USING ONLY THESE TWO STEPS, WHICH AL-KHWARIZMI CALLED "REBALANCING."






BEFORE GOING ON, LET ME SAY A FEW WORDS ABOUT THE LETTER MOST OFTEN USED AS A VARIABLE: **$x$**. THE POINT OF CHOOSING THIS MYSTERIOUS LETTER IS THAT IT STANDS FOR NOTHING IN PARTICULAR, WHETHER DISTANCE OR TIME OR PRICE. ALGEBRA, SAYS $x$, WORKS ON ANY VARIABLE, NO MATTER WHAT IT "MEANS." $x$ CAN BE ANYTHING!

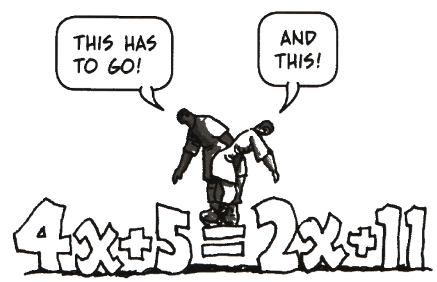



**Example 1.** SOLVE

$$4x + 5 = 2x + 11$$

$4x$ AND $2x$ ARE CALLED THE **VARIABLE TERMS,** WHILE THE "NAKED NUMBERS" 5 AND 11 ARE THE **CONSTANT TERMS.**


TO REBALANCE, WE CLEVERLY CHOOSE JUST THE RIGHT THINGS TO ADD OR SUBTRACT TO **REMOVE ALL VARIABLE TERMS** FROM THE **RIGHT** AND ALL **CONSTANT TERMS** FROM THE **LEFT.**




SUBTRACTING 5 WILL CLEAR THE CONSTANT FROM THE LEFT, AND SUBTRACTING $2x$ WILL CLEAR THE VARIABLE TERM FROM THE RIGHT. LET'S DO IT—TO BOTH SIDES!

$$\begin{array}{rcl} 4x + 5 & = & 2x + 11 \\ -5 & & -5 \\ -2x & & -2x \\ \hline 4x - 2x & = & 11 - 5 \\ 2x & = & 6 \end{array}$$

WE'RE ALMOST THERE! MULTIPLYING EVERYTHING BY 1/2, THE RECIPROCAL OF 2, WILL LEAVE $x$ ALONE ON THE LEFT SIDE AND SOLVE THE EQUATION.

$$2x/2 = 6/2$$
$$x = 3$$

FINALLY, WE PLUG $x = 3$ INTO THE ORIGINAL EQUATION TO CHECK THAT IT REALLY IS A SOLUTION.

$$4(3) + 5 \stackrel{?}{=} 2(3) + 11$$
$$12 + 5 \stackrel{?}{=} 6 + 11$$
$$17 = 17$$

Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.