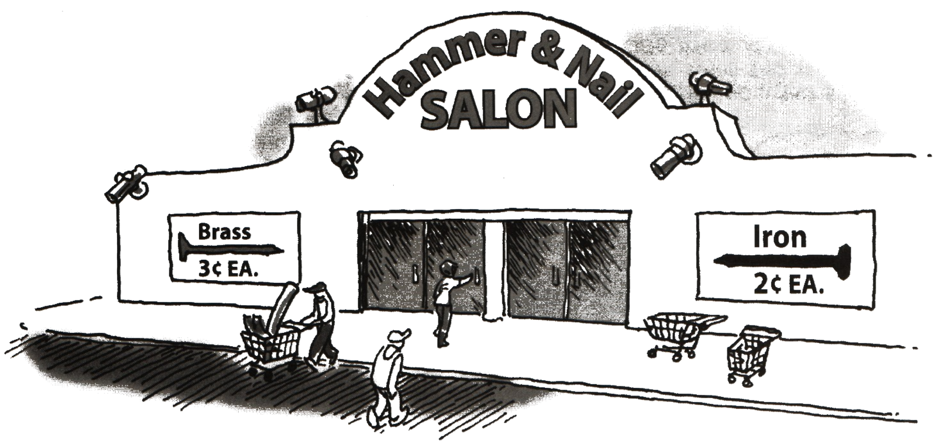
REALITY IS FULL OF VARIABLES. HEIGHTS AND WEIGHTS RISE AND FALL... PRICES GO UP AND (SOMETIMES) DOWN... THE WORLD IS ALWAYS CHANGING IN COUNTLESS WAYS... SO SHOULDN'T WE LET AT LEAST ONE MORE VARIABLE INTO OUR EQUATIONS AND GET A LITTLE MORE **REAL?**

LET'S START WITH A **CARPENTRY PROJECT.** CELIA GOES TO THE HARDWARE STORE FOR SOME NAILS. SHE NEEDS TWO DIFFERENT KINDS, BRASS AND IRON.



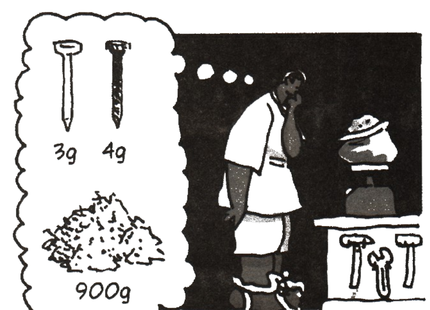
Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.

FOR SOME REASON OR OTHER, SHE TOSSES THEM ALL INTO THE SAME BAG...

AND TAKES THEM TO KEVIN'S WOOD SHOP.

KEVIN IS NOT HAPPY! HE WANTS TO KNOW HOW MANY NAILS **OF EACH KIND** CELIA BOUGHT!

KEVIN'S FIRST IDEA IS TO **WEIGH** THE NAILS. THE SCALE TELLS HIM THEY WEIGH 900 GRAMS. HE ALSO FINDS THAT ONE BRASS NAIL WEIGHS 3 GRAMS, WHILE ONE IRON NAIL WEIGHS 4 GRAMS.



NOW HE GETS ALGEBRAIC. HE LETS

B = THE NUMBER OF BRASS NAILS

I = THE NUMBER OF IRON NAILS

THEN $3B$ IS THE WEIGHT OF ALL THE BRASS NAILS, IN GRAMS, AND $4I$ IS THE WEIGHT OF ALL THE IRON NAILS, ALSO IN GRAMS. THE SUM OF THESE EXPRESSIONS IS THE TOTAL WEIGHT, 900 GRAMS, AND THIS STATEMENT BECOMES AN EQUATION.

$$3B + 4I = 900 \tag{1}$$

KEVIN TRIES SOLVING FOR B.

$$3B + 4I = 900 \quad \text{(EQUATION 1)}$$

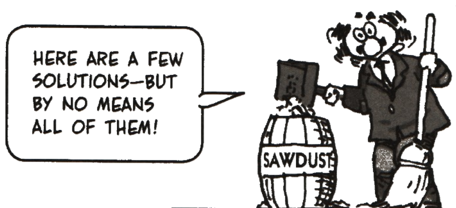
$$3B = 900 - 4I \quad \text{(SUBTRACTING } 4I \text{ FROM BOTH SIDES)}$$

$$(2) \quad B = 300 - \frac{4}{3}I \quad \text{(DIVIDING BOTH SIDES BY 3)}$$

INSTEAD OF FINDING A NUMBER, HE GETS AN EXPRESSION INVOLVING I. IN EQUATION (2), KEVIN HAS SOLVED FOR B "IN TERMS OF I." SO WHAT'S B? KEVIN AND CELIA ARE LEFT SCRATCHING THEIR HEADS.

IN FACT, **MANY** POSSIBLE VALUES OF B AND I SOLVE EQUATION 2. FOR INSTANCE, IF WE SIMPLY **GUESS** THAT $I = 30$, WE CAN PLUG THAT INTO (2) AND FIND

$$B = 300 - \frac{4}{3}(30)$$
$$= 300 - 40$$
$$= 260$$



I	B	$3B + 4I$
3	296	900
6	292	900
9	288	900
12	284	900
93	176	900
99	168	900
...	...	
200	$33\frac{1}{3}$	900

AND THIS PAIR OF VALUES, $I = 30$, $B = 260$, SOLVES EQUATION 1, AS WE SEE BY PLUGGING THEM IN.

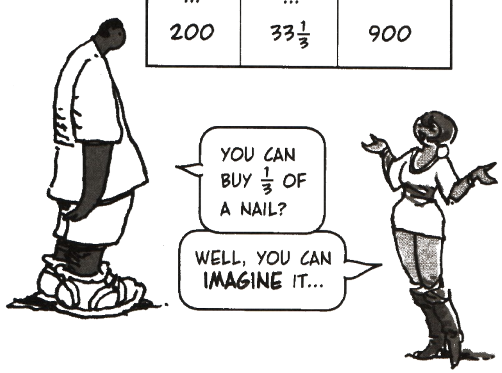
$$3(260) + 4(30)$$
$$= 780 + 120$$
$$= 900$$

IF WE HAD PICKED SOME OTHER VALUE OF I, SAY $I = 93$, THEN

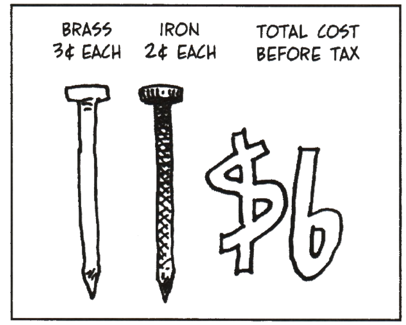
$$B = 300 - \frac{4}{3}(93)$$
$$B = 300 - 124 = 176$$

AND YOU CAN CHECK THAT THIS PAIR OF VALUES ALSO SOLVES EQUATION (1).

FOR ANY VALUE OF I, THERE IS A CORRESPONDING VALUE OF B. THE EQUATION HAS **MANY SOLUTIONS.**



CAN KEVIN EVER FIND B AND I USING ALGEBRA? JUST MAYBE... BECAUSE CELIA HAS SOME **MORE INFORMATION:** SHE REMEMBERS THE **COST** OF THE NAILS!



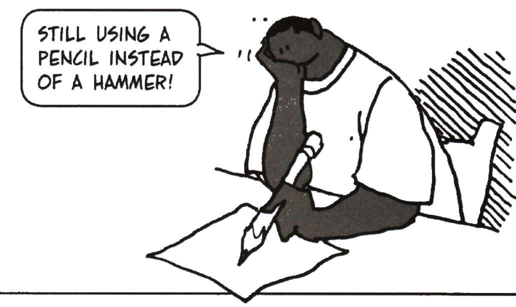
KEVIN SETS UP A NEW EQUATION. ALL NUMBERS ARE IN CENTS.

$3B$ = COST OF BRASS NAILS

$2I$ = COST OF IRON NAILS

THE TOTAL COST = 600 CENTS OR \$6.00, IS THE SUM OF THESE.

$$(3) \quad 3B + 2I = 600$$



THE FIRST EQUATION WITH TWO VARIABLES HAS MANY SOLUTIONS. CAN IT BE THAT A SECOND EQUATION WILL NARROW THE POSSIBILITIES TO A SINGLE PAIR OF NUMBERS, THE ACTUAL ANSWER? CAN WE FIND VALUES OF B AND I THAT SATISFY **BOTH** EQUATIONS **AT THE SAME TIME?**

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