TWO EQUATIONS IN TWO VARIABLES

START WITH TWO EQUATIONS LIKE THIS:

$$ax + by = e$$
$$cx + dy = f$$

IF I RUN OUT OF LETTERS, I'LL USE PIECES OF FRUIT!

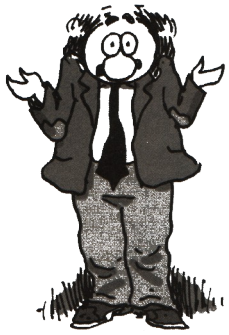
HERE a, b, c, d, e, AND f CAN BE ANY NUMBERS, AND x AND y ARE THE VARIABLES. NOTE THAT THERE ARE NO TERMS INVOLVING PRODUCTS OF VARIABLES LIKE xy OR xx OR x/y, JUST x AND y ALONE, WITH CONSTANT COEFFICIENTS. WE'VE JUST SEEN AN EXAMPLE: CELIA AND KEVIN'S CARPENTRY EQUATIONS (NOW USING x AND y INSTEAD OF B AND I).

$$3x + 4y = 900$$
$$3x + 2y = 600$$

WE WILL NOW SHOW THREE DIFFERENT WAYS TO SOLVE THIS PAIR OF EQUATIONS—THREE! AND THESE THREE WAYS ARE CALLED...

SUBSTITUTION, ELIMINATION,

AND, UM, WELL, THE THIRD WAY DOESN'T EXACTLY HAVE A NAME...



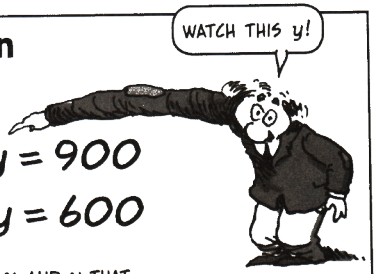
Substitution

AGAIN, START WITH THESE EQUATIONS:

(4) $3x + 4y = 900$

(5) $3x + 2y = 600$

WE SEEK VALUES OF x AND y THAT SOLVE BOTH EQUATIONS SIMULTANEOUSLY.



START BY USING EQUATION 5 TO SOLVE FOR y IN TERMS OF x.

(5) $3x + 2y = 600$

$2y = 600 - 3x$

(6) $y = 300 - \frac{3}{2}x$

SINCE y IS THE SAME AS THAT EXPRESSION IN x, WE CAN **SUBSTITUTE** IT FOR y IN EQUATION 4.

$$3x + 4\left(300 - \tfrac{3}{2}x\right) = 900$$

NOW WE HAVE AN EQUATION IN ONE VARIABLE, x ALONE.

$3x + 4(300 - \frac{3}{2}x) = 900$

$3x + 1200 - 6x = 900$

$6x - 3x = 1200 - 900$

$3x = 300$

$x = 100$

AND y? THE VERY FIRST STEP WAS TO FIND y IN TERMS OF x.

(6) $y = 300 - \frac{3}{2}x$

$y = 300 - \frac{3}{2}(100)$

$y = 300 - 150$

$y = 150$

SO THE ANSWER IS

$x = 100$ (BRASS NAILS)

$y = 150$ (IRON NAILS)

AND WE CHECK THAT THIS SOLUTION SOLVES **BOTH** EQUATIONS.

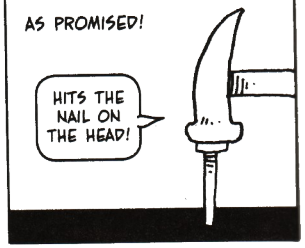
(4) $3(100) + 4(150) \stackrel{?}{=} 900$

$300 + 600 = 900$

(5) $3(100) + 2(150) \stackrel{?}{=} 600$

$300 + 300 = 600$

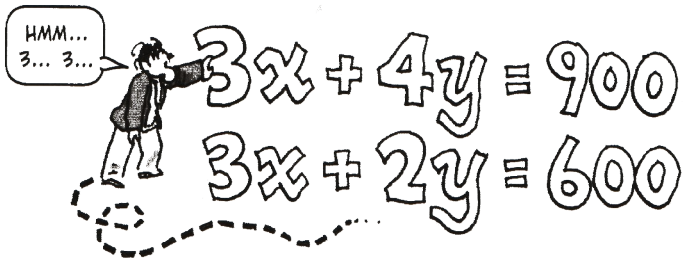
AS PROMISED!



Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.

Elimination

THE SUBSTITUTION METHOD GETS RID OF ("ELIMINATES") ONE VARIABLE IN A RATHER ROUNDABOUT WAY. THIS METHOD GOES STRAIGHT TO IT!



IF WE SUBTRACT THE LEFT SIDE OF ONE EQUATION FROM THE LEFT SIDE OF THE OTHER, THOSE $3x$ TERMS WILL CANCEL OUT. LET'S TRY THAT SUBTRACTION.

$$\begin{array}{r} 3x + 4y = 900 \\ -\,(3x + 2y = 600) \\ \hline 4y - 2y \rightarrow 2y = 300 \leftarrow 900 - 600 \end{array}$$

SUBTRACTING LEFT FROM LEFT AND RIGHT FROM RIGHT, WE'RE STILL TAKING AWAY EQUAL THINGS FROM EQUAL THINGS—SO THE RESULTS MUST BE EQUAL TOO!

NOW x IS ELIMINATED, AND WE HAVE FOUND

$$2y = 300$$

$$\boldsymbol{y = 150}$$

FIND x BY PLUGGING IN 150 FOR y IN EITHER OF THE ORIGINAL EQUATIONS.

(4) $3x + 4y = 900$

$3x + 4(150) = 900$

$3x + 600 = 900$

$3x = 300$

$\boldsymbol{x = 100}$

Method 3

THIS ONE MIGHT BE CALLED "FIND y TWICE." THE EQUATIONS

(4) $3x + 4y = 900$

(5) $3x + 2y = 600$

ALLOW US TO SOLVE FOR y IN TERMS OF x IN TWO DIFFERENT WAYS.

FROM (4):

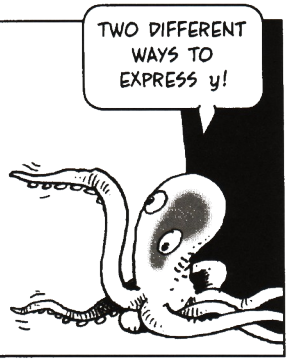
$3x + 4y = 900$

$4y = 900 - 3x$

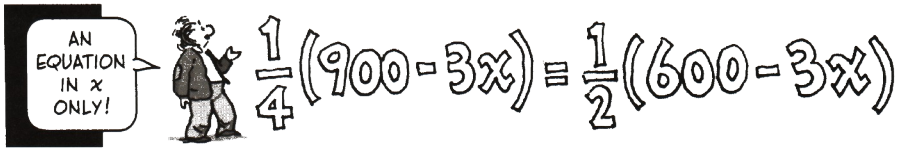
(7) $y = \frac{1}{4}(900 - 3x)$

AND FROM (5), WE ALREADY FOUND

(6) $y = \frac{1}{2}(600 - 3x)$



THE EXPRESSIONS $\frac{1}{4}(900-3x)$ AND $\frac{1}{2}(600-3x)$ ARE BOTH EQUAL TO y, SO THEY MUST BE EQUAL TO EACH OTHER.



WE SOLVE IT WITHOUT MUCH FUSS:

$\frac{1}{4}(900 - 3x) = \frac{1}{2}(600 - 3x)$

$900 - 3x = 2(600 - 3x)$ (MULTIPLYING BY 4 TO CLEAR FRACTIONS)

$900 - 3x = 1200 - 6x$

$6x - 3x = 1200 - 900$

$3x = 300$

$x = 100$

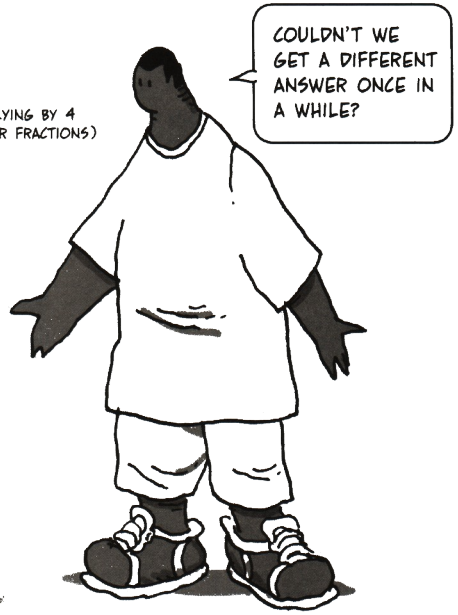
FIND y FROM EITHER (6) OR (7) BY PLUGGING IN 100 FOR x.

$y = \frac{1}{2}(600 - 3x)$

$y = \frac{1}{2}(600 - (3)(100))$

$y = \frac{1}{2}(300)$

$y = 150$



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