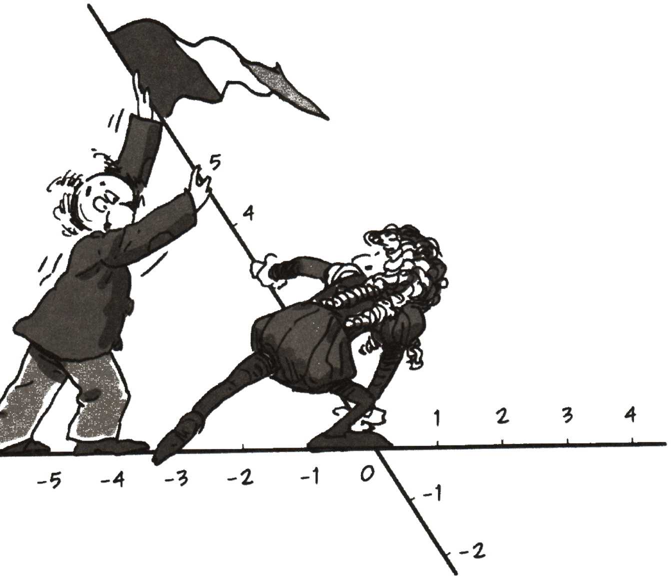
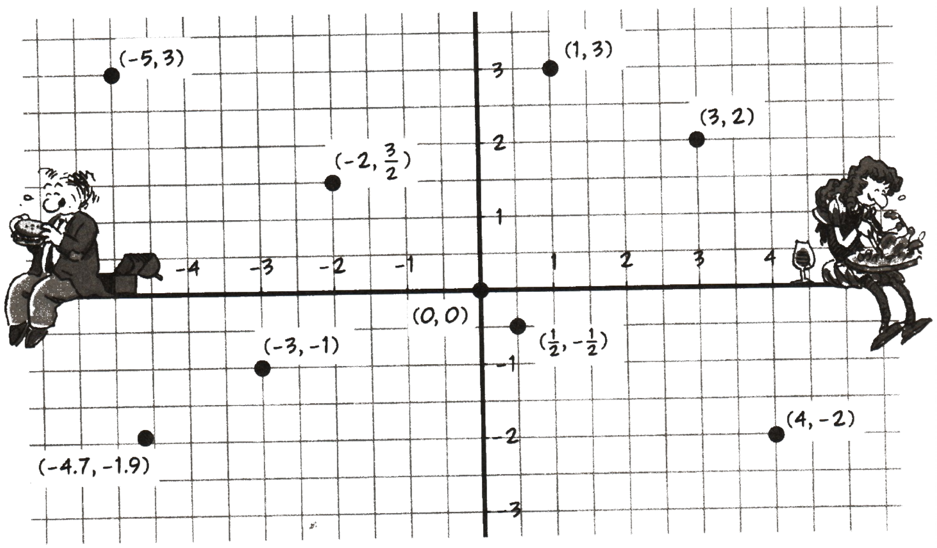
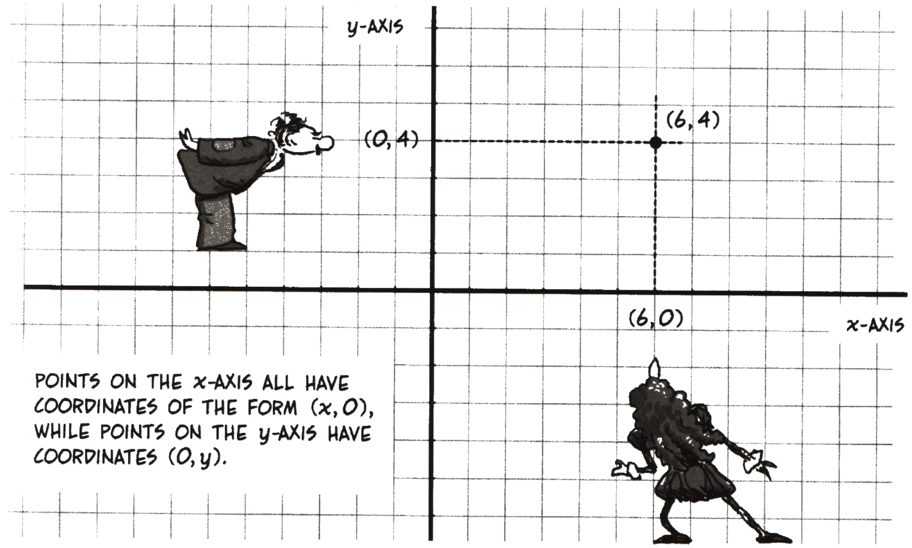
DESCARTES WANTED TO DRAW THE RELATIONSHIP BETWEEN TWO VARIABLES, SO INSTEAD OF ONE NUMBER LINE, HE DREW TWO... AND RATHER THAN RUNNING THEM SIDE BY SIDE, HE CROSSED THEM AT THEIR ZERO POINTS.



NOW THE WHOLE PLANE BECOMES A GRID. EVERY POINT IS PROVIDED WITH AN "ADDRESS" CONSISTING OF TWO NUMBERS IN ORDER, LIKE THIS: (x, y). THE FIRST NUMBER SAYS WHERE THE POINT IS HORIZONTALLY ON THE GRID, THE SECOND NUMBER WHERE IT IS VERTICALLY. THE POINT WHERE THE NUMBER LINES CROSS, CALLED THE **ORIGIN**, HAS ADDRESS $(0, 0)$.

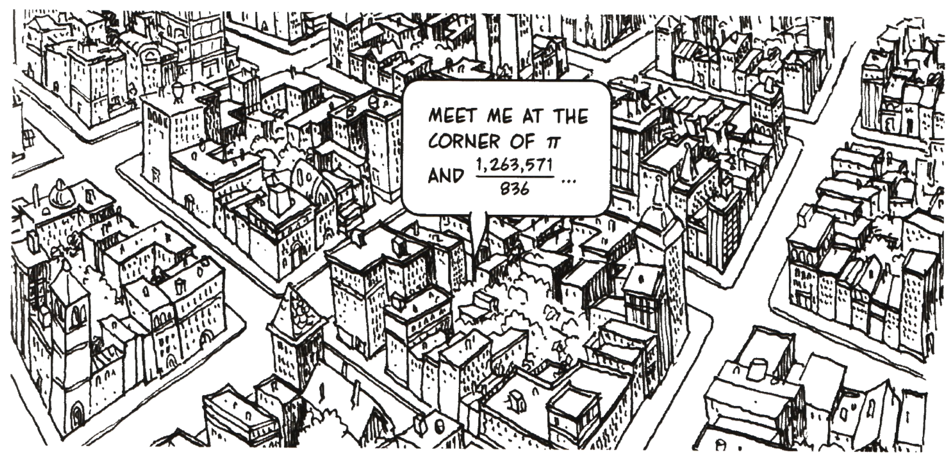


THE HORIZONTAL NUMBER LINE IS OFTEN CALLED THE **x-AXIS** AND THE VERTICAL NUMBER LINE THE **y-AXIS.** THE TWO NUMBERS OF A POINT'S ADDRESS ARE CALLED ITS **x-COORDINATE** AND ITS **y-COORDINATE.** TO FIND A POINT'S x-COORDINATE, FOLLOW A VERTICAL LINE FROM THE POINT TO THE x-AXIS; TO FIND ITS y-COORDINATE, GO HORIZONTALLY FROM THE POINT TO THE y-AXIS.



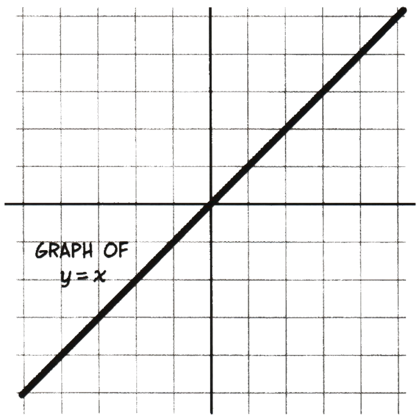
POINTS ON THE x-AXIS ALL HAVE COORDINATES OF THE FORM $(x, 0)$, WHILE POINTS ON THE y-AXIS HAVE COORDINATES $(0, y)$.

IF A CITY WERE LAID OUT LIKE THIS (AND MANY ARE—CHECK OUT A MAP OF NEW YORK CITY'S MANHATTAN), YOU MIGHT SAY THAT THE POINT (x, y) IS AT THE INTERSECTION OF x AVENUE AND y STREET. OF COURSE, OUR "CITY" HAS FRACTIONAL AND IRRATIONAL STREETS, TOO...



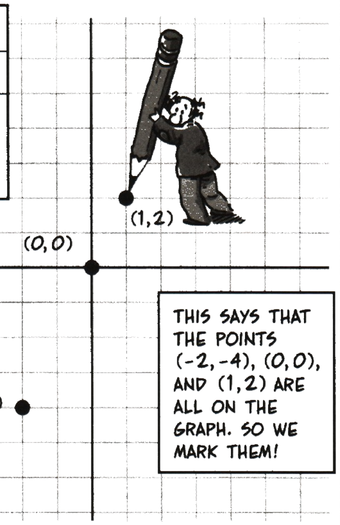
Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.

NOW LET'S PICTURE A SIMPLE EQUATION, $y = x$. A PAIR (x, y) SATISFIES THIS EQUATION WHEN THE TWO COORDINATES x AND y ARE **EQUAL** TO EACH OTHER. WE MARK ALL THE POINTS WHERE $x = y$, SUCH AS $(0, 0)$, $(1, 1)$, $(-3.14, -3.14)$, AND ALL THE REST. THESE POINTS LIE ON A STRAIGHT LINE CALLED THE EQUATION'S **GRAPH**.



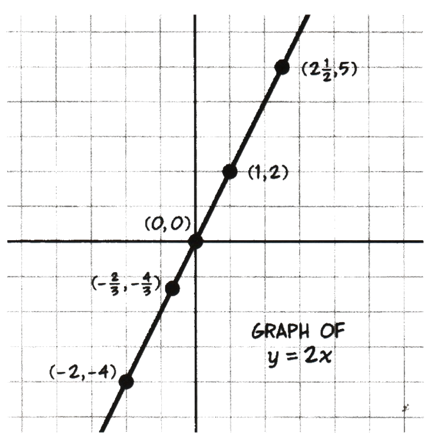
NEXT WE "GRAPH" THE EQUATION $y = 2x$. BEGIN BY FILLING IN A SMALL TABLE WITH A FEW VALUES OF x AND y. WE CAN PICK ANY OLD VALUES OF x; IT DOESN'T MATTER.

$y = 2x$	
x	y
-2	-4
0	0
1	2

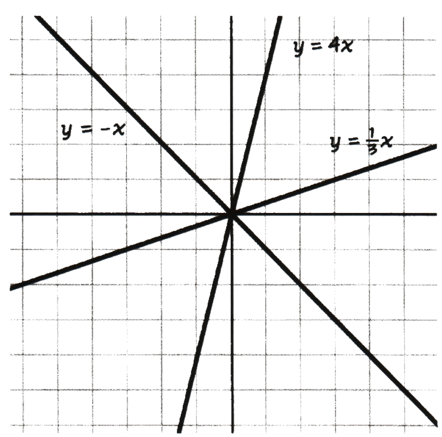


THIS SAYS THAT THE POINTS $(-2,-4)$, $(0,0)$, AND $(1,2)$ ARE ALL ON THE GRAPH. SO WE MARK THEM!

IF YOU LAY A STRAIGHTEDGE AGAINST THOSE POINTS, YOU'LL FIND THAT THEY LIE ON A STRAIGHT LINE. **EVERY POINT ON THE LINE** SATISFIES THE EQUATION $y = 2x$. THIS LINE IS THE GRAPH OF $y = 2x$. AS YOU SEE, IT'S STEEPER THAN THE GRAPH OF $y = x$.

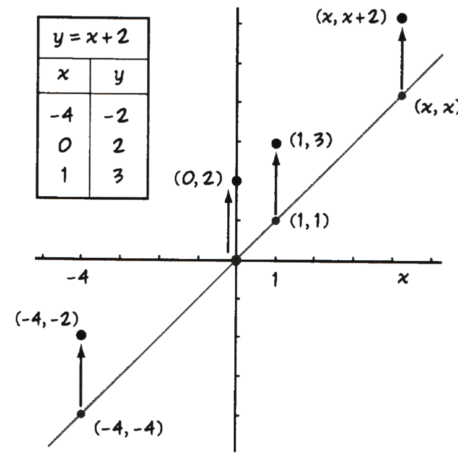


HERE ARE THE GRAPHS OF EQUATIONS $y = mx$ FOR SEVERAL VALUES OF m. ALL OF THEM PASS THROUGH THE ORIGIN (WHY?), AND LARGER VALUES OF m MAKE STEEPER LINES. WHEN m IS NEGATIVE, THE GRAPH SLOPES "BACKWARD," I.E., IT GOES DOWN TO THE RIGHT.

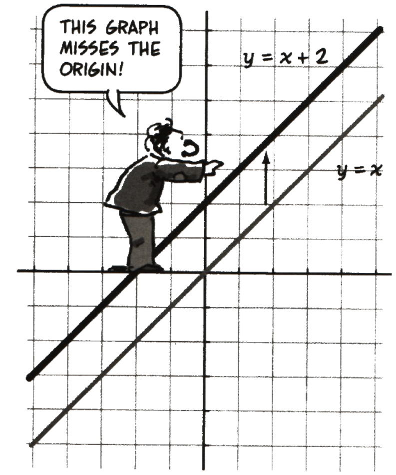


HOW ABOUT THE EQUATION $y = x + 2$? GIVEN ANY VALUE OF x, ADD 2 TO FIND y. STARTING AT ANY POINT x ON THE x-AXIS, GO x UNITS VERTICALLY, THEN UP TWO UNITS.

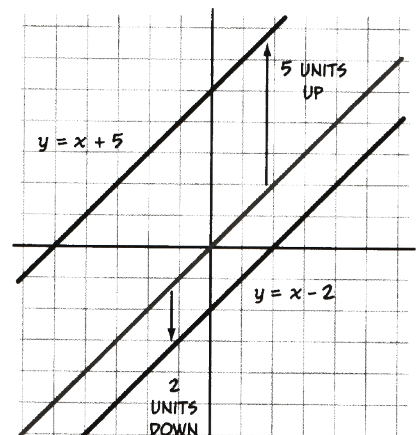
$y = x + 2$	
x	y
-4	-2
0	2
1	3



I HOPE YOU CAN SEE THAT THIS GRAPH LOOKS EXACTLY LIKE THE GRAPH OF THE EQUATION $y = x$, BUT SHIFTED TWO UNITS UPWARD.



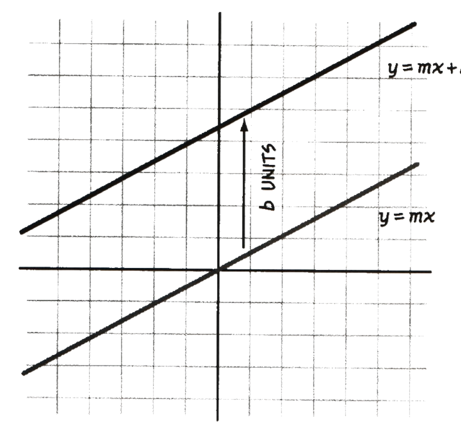
GIVEN ANY NUMBER a, THE GRAPH OF $y = x + a$ LOOKS LIKE THE GRAPH OF $y = x$ SHIFTED VERTICALLY a UNITS (UP IF $a > 0$, DOWN IF $a < 0$).



IN THE SAME WAY, IF m IS ANY NUMBER, THE GRAPH OF

$$y = mx + b$$

IS IDENTICAL TO THE GRAPH OF $y = mx$, BUT SHIFTED b UNITS VERTICALLY.



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