

Parallel Lines

AS WE SAW IN THE LAST CHAPTER, A PAIR OF LINEAR EQUATIONS MAY HAVE **NO SOLUTION**... AND NOW WE CAN SEE WHY. THIS CAN HAPPEN ONLY WHEN THE TWO EQUATIONS' GRAPHS **NEVER CROSS**.

AND IF THE LINES **DON'T CROSS**?

LIKE, NEVER?

NEVER EVER?

TWO LINES THAT NEVER CROSS ARE CALLED **PARALLEL LINES**, AND AS YOU CAN SEE, PARALLEL LINES HAVE THE SAME **SLOPE**.

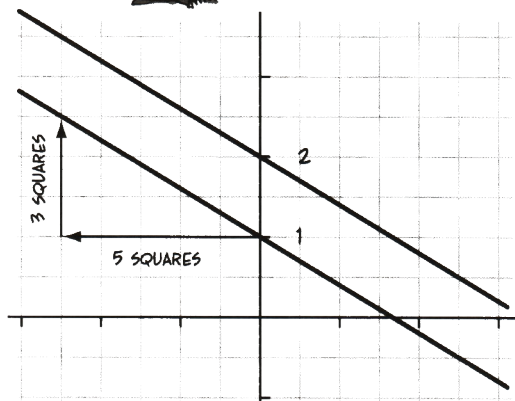
WE CAN EASILY SEE WHETHER A PAIR OF LINEAR EQUATIONS HAVE PARALLEL GRAPHS BY PUTTING THE EQUATIONS INTO SLOPE-INTERCEPT FORM. FOR EXAMPLE:

$$\begin{aligned}(1) \quad & 3x + 5y = 5 \\ (2) \quad & 6x + 10y = 20\end{aligned}$$

IN POINT-SLOPE FORM, THESE BECOME

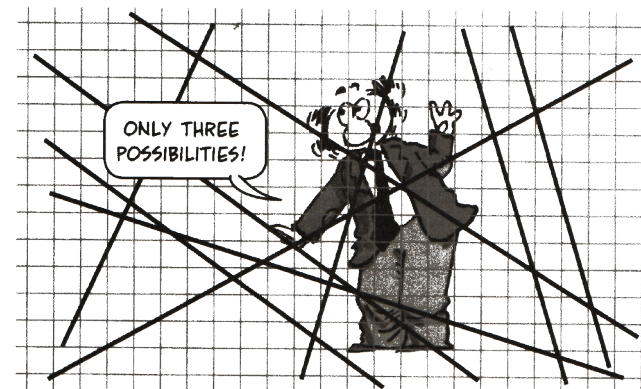
$$\begin{aligned}(1a) \quad & y = -\frac{3}{5}x + 1 \\ (2a) \quad & y = -\frac{3}{5}x + 2\end{aligned}$$

THE **y-INTERCEPTS** ARE DIFFERENT, SO THE GRAPHS ARE TWO SEPARATE LINES, BUT THE **SLOPES** ARE THE SAME, $-\frac{3}{5}$, SO THE LINES ARE **PARALLEL**. THE EQUATIONS HAVE **NO COMMON SOLUTION**.



GIVEN TWO LINEAR EQUATIONS, EITHER:

1. THEY HAVE THE **SAME GRAPH**;
2. THEIR GRAPHS ARE **PARALLEL**;
3. THEIR GRAPHS **CROSS** AT A SINGLE POINT.



GIVEN TWO EQUATIONS, HOW CAN WE KNOW WHICH OPTION HOLDS TRUE? LET'S SUPPOSE a, b, c, d, e , AND f ARE SOME FIXED NUMBERS, AND THAT NEITHER b NOR d IS ZERO. (b AND d WILL BE THE COEFFICIENTS OF y IN THE TWO EQUATIONS, AND WE'LL WANT TO DIVIDE BY THEM.) WE WANT TO KNOW ABOUT THE STANDARD-FORM EQUATIONS 3 AND 4.

$$(3) \quad ax + by = e$$

WE NEXT PUT THEM IN POINT-SLOPE FORM:

$$(4) \quad cx + dy = f$$

$$\begin{aligned}(3a) \quad y &= -\frac{a}{b}x + \frac{e}{b} \\ (4a) \quad y &= -\frac{c}{d}x + \frac{f}{d}\end{aligned}$$

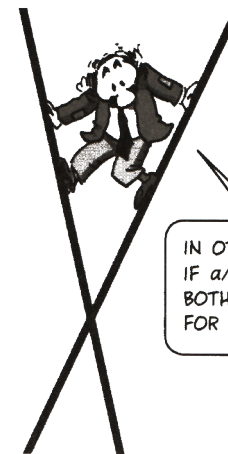
← y-INTERCEPTS
← SLOPES

COMPARE SLOPES AND INTERCEPTS, AND DRAW THIS...



CONCLUSION:

1. IF $a/b = c/d$ AND $e/b = f/d$, THEN THE EQUATIONS' GRAPHS HAVE THE SAME SLOPE AND **y-INTERCEPT**. THEY ARE THE SAME LINE! EVERY POINT ON THIS LINE SOLVES BOTH EQUATIONS.
2. IF $a/b = c/d$ AND $e/b \neq f/d$, THEN THE GRAPHS HAVE THE SAME SLOPE BUT DIFFERENT **y-INTERCEPTS**. THEY ARE PARALLEL LINES, AND THERE IS NO SOLUTION.
3. IF $a/b \neq c/d$, THEN THE EQUATIONS HAVE DIFFERENT SLOPES. THEIR GRAPHS **CROSS** AT A SINGLE POINT, WHICH SOLVES BOTH EQUATIONS SIMULTANEOUSLY.



Horizontal and Vertical Lines

IN THE EQUATION

$$ax + by = c$$

WE KEEP ASSUMING $b \neq 0$. THIS MEANS WE'VE LOOKED ONLY AT EQUATIONS LIKE

$$2x + 6y = 4$$

$$9x - 503y = 7,021,077$$

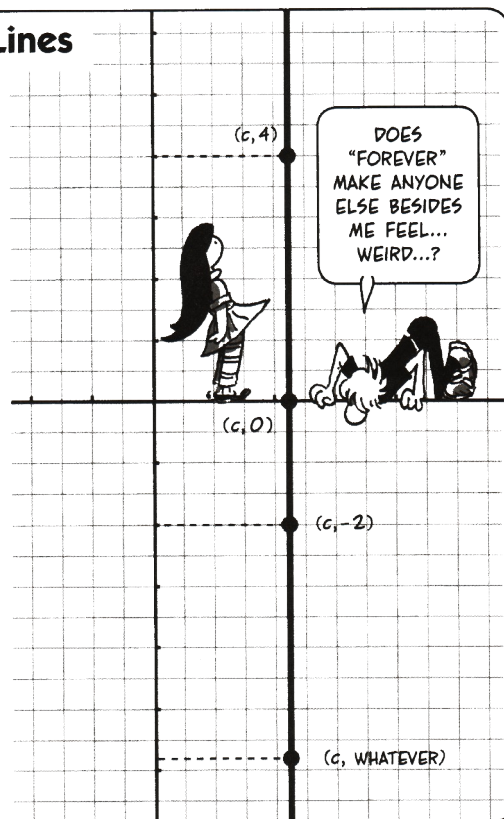
OR EVEN

$$y = 8$$

(a , THE COEFFICIENT OF x , CAN BE ZERO!) BUT WHAT ABOUT WHEN $b = 0$? THESE ARE EQUATIONS LIKE

$$x = c$$

WHICH HAS A **VERTICAL GRAPH**. IT'S ALL THE POINTS WITH AN x -COORDINATE EQUAL TO c . A VERTICAL LINE'S SLOPE IS... **INFINITE**. IT RISES (AND FALLS) FOREVER WITH NO RUN AT ALL.

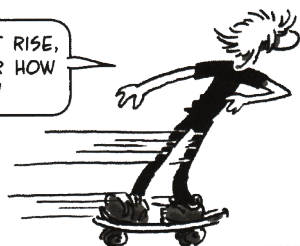


WHEN $a = 0$, ON THE OTHER HAND, THE EQUATION LOOKS LIKE

$$y = c$$

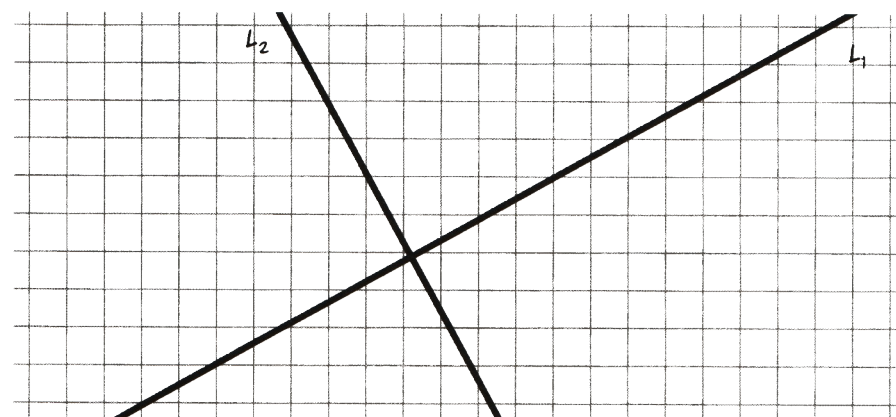
ITS GRAPH IS **HORIZONTAL**, A LINE OF SLOPE **ZERO** (NO RISE, ALL RUN).

IT DOESN'T RISE, NO MATTER HOW FAR I RUN!

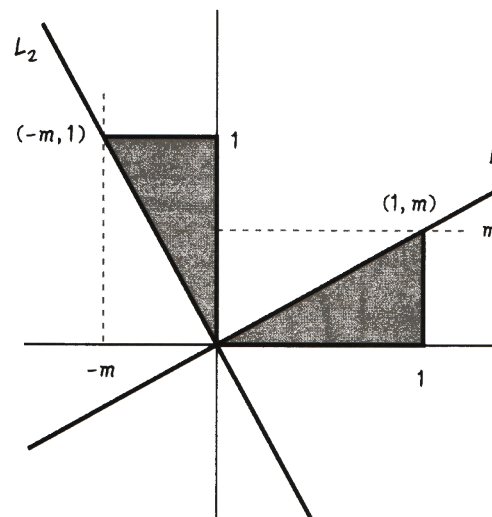


Perpendicular Lines

FINALLY, JUST FOR FUN, LET'S SEE WHAT IT MEANS ALGEBRAICALLY FOR TWO LINES TO MEET AT A RIGHT ANGLE. SUCH LINES ARE CALLED **PERPENDICULAR**. AS YOU GO AROUND THEIR INTERSECTION, ALL FOUR ANGLES ARE EQUAL, LIKE THE CORNERS OF A SQUARE. THE COORDINATE AXES ARE AN EXAMPLE. GIVEN TWO PERPENDICULAR LINES L_1 AND L_2 , LET'S SUPPOSE THAT L_1 HAS SLOPE m . WHAT'S THE SLOPE OF L_2 IN TERMS OF m ?



FIRST, SLIDE THE TWO LINES SO THAT THEY CROSS AT THE ORIGIN. THE SLOPES REMAIN THE SAME, AND THE LINE L_1 NOW CONTAINS THE POINT $(1, m)$. (THE LINE $y = mx$ ALWAYS CONTAINS THE POINT $(1, m)$!)



IN THE DIAGRAM, THE TWO GRAY TRIANGLES ARE EXACTLY THE SAME, ONLY TURNED. EACH HAS A SIDE OF LENGTH 1 AND A SIDE OF LENGTH m . SO L_2 CONTAINS THE POINT $(-m, 1)$.

THIS MEANS THAT L_2 HAS SLOPE

$$\frac{1}{-m} = -\frac{1}{m}$$

PERPENDICULAR LINES HAVE SLOPES THAT ARE EACH OTHER'S **NEGATIVE RECIPROCAL** (AS LONG AS NEITHER LINE IS VERTICAL!).