What you should learn:

Goal 1 How to simplify a rational expression

Goal 2 How to use rational expressions as real-life models

Why you should learn it:

You can use rational expressions as models of real-life situations such as in relating air pressure to altitude.

Goal 1 Simplifying a Rational Expression

A fraction whose numerator and denominator are polynomials is called a **rational expression.** Some examples are

$$\frac{3}{x+4}, \quad \frac{2x}{x^2-9}, \quad \text{and} \quad \frac{6}{x^2+1}.$$

The **domain** of a rational expression is the set of all real numbers *except* those for which the denominator is zero. For instance, the domain of the first expression is all real numbers except -4.

A rational expression is **simplified** or **in reduced form** if its numerator and denominator have no factors in common (other than ± 1). To reduce fractions, apply the following property. Be sure you see that this property allows you only to divide out factors, *not* terms.

Simplifying Fractions

Let a, b, and c be nonzero real numbers.

$\frac{a\cancel{c}}{b\cancel{c}} = \frac{a}{b}$ Divide out common factor of c.

Example 1 *When and When Not to Divide*

a. $\frac{\cancel{2} \cdot x}{\cancel{2}(x+5)} = \frac{x}{x+5}$ — *You CAN divide out the common factor 2.*

b. $\frac{3+x}{3+2x}$ — *You CANNOT divide out the common term 3.*

c. $\frac{\cancel{x}(x^2+6)}{\cancel{x} \cdot x} = \frac{x^2+6}{x}$ — *You CAN divide out the common factor x.*

d. $\frac{x+4}{x}$ — *You CANNOT divide out the common term x.*

Simplifying a rational expression usually requires two steps. First, factor the numerator and denominator. Then, divide out any *factors* that are common to both the numerator and denominator. Your success in simplifying a rational expression lies in your ability to factor its numerator and denominator.

Excerpts from Larson, Roland (1998). *Algebra: An Integrated Approach*. Evanston, IL. McDougal LIttell.

Example 2 *Simplifying a Rational Expression*

Simplify $\frac{2x^2-6x}{6x^2}$.

Solution

$$\frac{2x^2-6x}{6x^2} = \frac{2x(x-3)}{6x^2} \qquad \textit{Factor numerator and denominator.}$$

$$= \frac{\cancel{2x}(x-3)}{\cancel{2x}(3x)} \qquad \textit{Divide out common factor } 2x.$$

$$= \frac{x-3}{3x} \qquad \textit{Simplified form}$$

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Goal 2 Using Rational Expressions in Real Life

Example 3 *Areas of Science Laboratories*

Rockville High School is building a new science wing. The architect has allocated spaces of x feet by x feet for both the biology and chemistry labs. At the request of the faculty, the principal agrees to increase the length of each lab by 5 feet and increase the width of the chemistry lab by 10 feet. Find an expression for the ratio of the area of the revised chemistry lab to the area of the revised biology lab. Evaluate the ratio when $x = 25$ feet and when $x = 30$ feet.

Problem Solving
Draw a Diagram

Solution

$$\text{Ratio} = \frac{\text{Area of Chemistry Lab}}{\text{Area of Biology Lab}}$$

$$= \frac{(x+5)(x+10)}{x(x+5)}$$

$$= \frac{x+10}{x} \qquad \textit{Divide out common factor.}$$

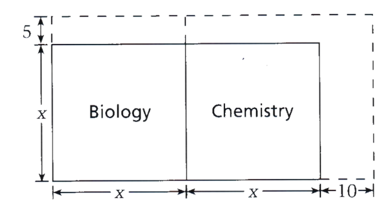
When $x = 25$ feet, the ratio is

$$\frac{25+10}{25} = \frac{35}{25} = 1.4$$

which means that the chemistry lab would be 40% larger than the biology lab. When $x = 30$ feet, the ratio is

$$\frac{30+10}{30} = \frac{40}{30} \approx 1.33$$

which means that the chemistry lab would be about 33% larger than the biology lab.



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What you should learn:

Goal 1 How to multiply and divide rational expressions

Goal 2 How to use rational expressions in real-life settings

Why you should learn it:

You can use rational expressions to model real-life situations, such as relating boiling points to altitudes.

Goal 1 Working with Rational Expressions

The rule for multiplying rational expressions is the same as the rule for multiplying numerical fractions. That is, *multiply numerators, multiply denominators, and write the new fraction in reduced form.*

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

To divide one rational expression by another, multiply the first by the reciprocal of the second. Be careful not to confuse the operation. Invert the divisor expression only.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Example 1 *Multiplying Rational Expressions*

Multiply: $\frac{4x^3}{3x} \cdot \frac{-6x^2}{10x^4}$

Solution

$$\frac{4x^3}{3x} \cdot \frac{-6x^2}{10x^4} = \frac{-24x^5}{30x^5}$$ *Multiply numerators and denominators.*

$$= \frac{-4 \cdot \cancel{6} \cdot \cancel{x^5}}{5 \cdot \cancel{6} \cdot \cancel{x^5}}$$ *Factor, and divide out common factors.*

$$= -\frac{4}{5}$$ *Simplified form*

Example 2 *Multiplying Rational Expressions*

Multiply: $\frac{x}{5x^2 - 20x} \cdot \frac{x-4}{2x^2 + x - 3}$

Solution

$$\frac{x}{5x^2 - 20x} \cdot \frac{x-4}{2x^2 + x - 3}$$
$$= \frac{x(x-4)}{(5x^2 - 20x)(2x^2 + x - 3)}$$
$$= \frac{x(x-4)}{5x(x-4)(x-1)(2x+3)}$$
$$= \frac{\cancel{x}\cancel{(x-4)}}{5\cancel{x}\cancel{(x-4)}(x-1)(2x+3)}$$
$$= \frac{1}{5(x-1)(2x+3)}$$

Excerpts from Larson, Roland (1998). *Algebra: An Integrated Approach*. Evanston, IL. McDougal LIttell.

To multiply a rational expression by a polynomial, rewrite the polynomial as a fraction whose denominator is 1. This helps you to avoid confusion in simplifying the product.

Example 3 *Multiplying a Rational Expression by a Polynomial*

Multiply: $\frac{3x}{2x^2 - 9x + 10} \cdot (2x - 5)$

Solution

$$\frac{3x}{2x^2 - 9x + 10} \cdot (2x - 5) = \frac{3x}{2x^2 - 9x + 10} \cdot \frac{2x-5}{1}$$
$$= \frac{3x(2x-5)}{(2x-5)(x-2)}$$
$$= \frac{3x\cancel{(2x-5)}}{\cancel{(2x-5)}(x-2)}$$
$$= \frac{3x}{x-2}$$

Example 4 *Dividing Rational Expressions*

Divide: $\frac{3x}{x+2} \div \frac{x-6}{x+2}$

Solution

$$\frac{3x}{x+2} \div \frac{x-6}{x+2} = \frac{3x}{x+2} \cdot \frac{x+2}{x-6}$$ *Multiply by reciprocal.*

$$= \frac{(3x)(x+2)}{(x+2)(x-6)}$$ *Multiply numerators and denominators.*

$$= \frac{(3x)\cancel{(x+2)}}{\cancel{(x+2)}(x-6)}$$ *Divide out common factors.*

$$= \frac{3x}{x-6}$$ *Simplified form*

Example 5 *Dividing a Rational Expression by a Polynomial*

Divide: $\frac{x^2 - 4}{2x^2} \div (x - 2)$

Solution

$$\frac{x^2-4}{2x^2} \div (x-2) = \frac{x^2-4}{2x^2} \cdot \frac{1}{x-2}$$ *Multiply by reciprocal.*

$$= \frac{(x+2)(x-2)}{2x^2(x-2)}$$ *Multiply numerators and denominators.*

$$= \frac{(x+2)\cancel{(x-2)}}{2x^2\cancel{(x-2)}}$$ *Divide out common factors.*

$$= \frac{x+2}{2x^2}$$ *Simplified form*