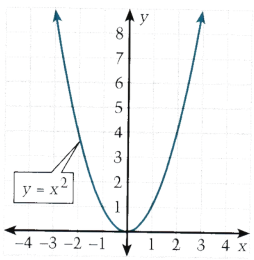
What you should learn:

Goal 1 How to identify functions and use function notation

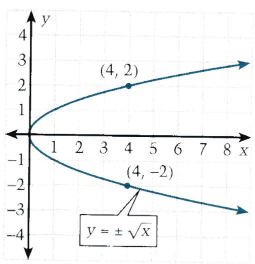
Goal 2 How to identify real-life relations that are functions

Why you should learn it:

Functions form a foundation for the study of advanced algebra and trigonometry.



y is a function of x.



y is not a function of x.

Goal 1 Identifying Functions

A **relation** is a set of ordered pairs (x, y). A **function** of x is a relation in which no two ordered pairs have the same x-value. You can identify a function by checking its graph. The x-values make up the **domain** (or input) of the function. The y-values are the **range** (or output). There are many ways to define a function: by an equation, a table, a graph, a sentence, or a set of ordered pairs.

LESSON INVESTIGATION

Investigating Functions

Partner Activity Graph each relation and decide whether it is a function of x. Explain your reasoning. If the relation is a function, state its domain and range.

a. $\{(0, 1), (1, 2), (1, 3), (2, 4), (3, 5), (4, 5)\}$

b. $\{(0, 3), (1, 3), (2, 3), (3, 3), (4, 3), (5, 3)\}$

c. $\{(0, 3), (1, 2), (2, 1), (3, 0), (4, 1), (5, 2), (6, 3)\}$

Example 1 *Identifying Relations That Are Functions*

Let $A = \{a, b, c\}$ and let $B = \{1, 2, 3, 4, 5\}$. Which of the following are functions relating the set A to the set B?

a. $\{(a, 1), (b, 4), (c, 4)\}$

b. $\{(a, 1), (b, 2), (c, 3), (b, 4), (a, 5)\}$

c.

Input value	a	b	c	b	a
Output value	1	2	3	4	5

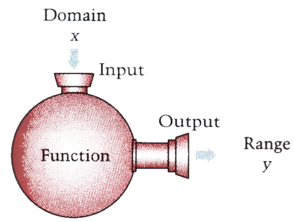
Solution

a. This set of ordered pairs *does* define a function relating A to B. No two ordered pairs have the same first coordinate.

b. This set of ordered pairs *does not* define a function relating A to B. Two of the pairs have the same first coordinate.

c. This is another way of representing the pairs in Part b. ■

Excerpts from Larson, Roland (1998). *Algebra: An Integrated Approach*. Evanston, IL. McDougal LIttell.



Study Tip

In a function, x is the independent variable and y is the dependent variable. In other words, if someone tells you the x-value, then you can find the y-value, because y depends on x. The reverse is not true. For instance, in Example 2 if someone tells you that the y-value is −3, then you cannot determine what the x-value is. Can you see why?

The terms *input* and *output* are used because a function can be thought of as a "machine" that changes numbers or other quantities. The input is put into the "function machine" and the function produces an output.

When a function is defined by an equation, it is convenient to name the function. For example, the set of ordered pairs (x, y) that satisfy the equation $y = 3x + 2$ form a function. By naming the function "f," you can use **function notation.**

***x-y* Notation**	**Function Notation**
(x, y) is a solution of $y = 3x + 2$.	$(x, f(x))$ is a solution of $f(x) = 3x + 2$.

The symbol $f(x)$ replaces y and is read as "the value of f at x" or simply as "f of x." When we evaluate $3x + 2$ for a particular value of x, we say we are evaluating f at x. Watch how this notation "works."

***x*-value**	**Value of *y*, or *f*(*x*)**
$x = 1$	$f(1) = 3(1) + 2 = 3 + 2 = 5$
$x = 2$	$f(2) = 3(2) + 2 = 6 + 2 = 8$

Up to this point in the text, we have been using parentheses to represent multiplication. The function notation $f(x)$ is a different use of parentheses. Remember that $f(x)$ means the value of f at x. It *does not* mean f times x.

Example 2 *Evaluating a Function*

Evaluate the function $f(x) = x^2 + 2x - 3$ at the given x-values.

a. $x = -2$ **b.** $x = 0$

Solution

a. To find the value of $f(x)$ when $x = -2$, substitute -2 for x in the equation that defines the function.

$$
\begin{aligned}
f(x) &= x^2 + 2x - 3 && \textit{Equation for } f\\
f(-2) &= (-2)^2 + 2(-2) - 3 && \textit{Substitute } -2 \textit{ for } x.\\
&= 4 - 4 - 3 && \textit{Simplify.}\\
&= -3 && \textit{Simplify.}
\end{aligned}
$$

b. To find the value of $f(x)$ when $x = 0$, substitute 0 for x in the equation that defines the function.

$$
\begin{aligned}
f(x) &= x^2 + 2x - 3 && \textit{Equation for } f\\
f(0) &= (0)^2 + 2(0) - 3 && \textit{Substitute 0 for } x.\\
&= 0 + 0 - 3 && \textit{Simplify.}
\end{aligned}
$$

What you should learn:

Goal 1 How to transform the graph of a function

Goal 2 How to use exponential functions as models of real-life problems

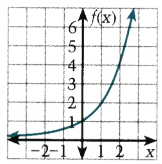
Why you should learn it:

You can model many real-life situations using exponential functions, such as college enrollments.

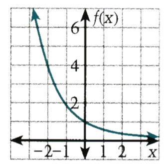
Goal 1 Transforming a Graph

In Chapter 8, you were introduced to graphs that model the exponential function.

Exponential Growth: $f(x) = 2^x$



Exponential Decay: $f(x) = \left(\frac{1}{2}\right)^x$



If the graph of a function is shifted about on the coordinate plane, it may then define a different function that is related in a simple way to the original function. Such a shift is one example of a **transformation.** In this lesson, you will learn to graph a function by transforming the graph of a simpler, related function.

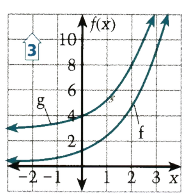
In Example 1, note how letters other than f, such as g and h, are commonly used to denote functions.

Example 1 *Vertical Shifts*

a. To sketch the graph of

$g(x) = 2^x + 3$

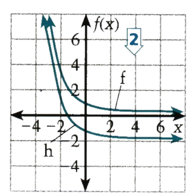
shift the graph of $f(x) = 2^x$ *three units up.*



b. To sketch the graph of

$h(x) = \left(\frac{1}{2}\right)^x - 2$

shift the graph of $f(x) = \left(\frac{1}{2}\right)^x$ *two units down.*

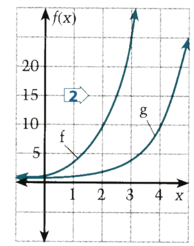


Example 2 *Horizontal Shifts*

a. To sketch the graph of

$g(x) = 3^{x-2}$

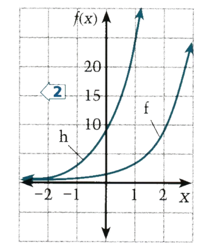
shift the graph of $f(x) = 3^x$ *two units to the right.*



b. To sketch the graph of

$h(x) = 3^{x+2}$

shift the graph of $f(x) = 3^x$ *two units to the left.*



One graph is a **reflection** of another graph in the x-axis if the two graphs are mirror images of each other. (The mirror lies on the x-axis.)

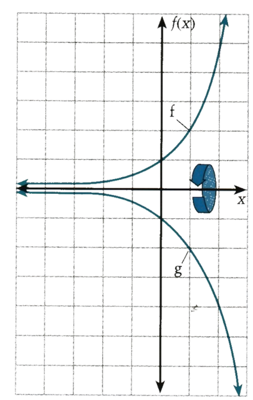
Lake LaCross, Mt. Steel, Olympic National Park

Example 3 *Reflections*

a. To sketch the graph of

$g(x) = -2^x$

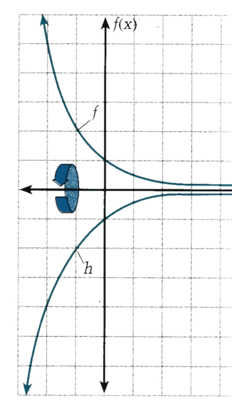
reflect the graph of $f(x) = 2^x$ in the x-axis.



b. To sketch the graph of

$h(x) = -\left(\frac{1}{2}\right)^x$

reflect the graph of $f(x) = \left(\frac{1}{2}\right)^x$ in the x-axis.



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