What you should learn:

Goal 1 How to construct a stem-and-leaf plot for data

Goal 2 How to construct a box-and-whisker plot for data

Why you should learn it:

Stem-and-leaf plots can help you organize large sets of data, such as the salaries of professional athletes.

Real Life Golf

Winnings in 1000's	
George Archer	$640
Bob Charles	$460
Charles Coody	$670
Bruce Crampton	$420
Jim Dent	$640
Dale Douglass	$450
Al Gelberger	$310
Harold Henning	$340
Dave Hill	$320
Mike Hill	$680
Rives McBee	$430
Jack Nicklaus	$340
Gary Player	$490
Chi Chi Rodriguez	$630
Lee Trevino	$950

Goal 1 Constructing Stem-and-Leaf Plots

A **stem-and-leaf plot** is a technique for ordering data in increasing order or decreasing order. In the following example, the *stem* consists of the tens digits for the data and the *leaves* consist of the units digits.

Unordered Data	Stem	Leaves	Ordered Data
52, 35, 26, 7, 31,	6	0 3 2 4	64, 63, 62, 60
31, 60, 14, 24, 54,	5	2 4 8 0	58, 54, 52, 50
15, 19, 24, 63, 58,	4	1 4 7 5 2	47, 45, 44, 42, 41
41, 23, 39, 8, 12,	3	5 1 1 9 4	39, 35, 34, 31, 31
18, 34, 62, 6, 44,	2	6 4 4 3 6 9 8	29, 28, 26, 26, 24, 24, 23
47, 45, 26, 29, 50,	1	4 5 9 2 8 7 1	19, 18, 17, 15, 14, 12, 11
17, 11, 64, 28, 42	0	7 8 6	8, 7, 6

Stem-and-Leaf Plot

Example 1 *Using a Stem-and-Leaf Plot*

The top 15 PGA Senior Tour golf winners for 1990 (through November 1) are shown in the table. Use a stem-and-leaf plot to order the winnings. *(Source:* USA Today*)*

Solution To construct a stem-and-leaf plot, consider only the first two digits of each number.

9	5
6	4 7 4 8 3
4	6 2 5 3 9
3	1 4 2 4

Key: 9 | 5 represents $950,000.

Now, using the stem-and-leaf plot, order the winnings (in thousands) as follows.

$950 Trevino	$680 M. Hill	$670 Coody	$640 Archer	$640 Dent
$630 Rodriguez	$490 Player	$460 Charles	$450 Douglass	$430 McBee
$420 Crampton	$340 Henning	$340 Nicklaus	$320 D. Hill	$310 Gelberger ■

Excerpts from Larson, Roland (1998). *Algebra: An Integrated Approach*. Evanston, IL. McDougal LIttell.

Goal 2 Constructing Box-and-Whisker Plots

You already know how to find the *average*, or **mean**, of n numbers: Add the numbers and divide the sum by n. It is one of the ways to represent the "middle" or "center" of a collection of numbers. (We will say more about measures of *central tendency* in the next lesson.) Another way to represent the middle of a collection of numbers is with a number called the *median*.

The Median of a Collection of Numbers

1. The **median** of an odd collection of numbers, arranged *in order*, is the middle number.
2. The **median** of an even collection of numbers, arranged *in order*, is the average of the two middle numbers.

Example 2 *Finding the Median*

a. The median of 2, 4, 4, 5, $\underbrace{6}_{\text{Middle}}$, 8, 8, 9, 10 is 6.

b. The median of 1, 1, 3, 4, 4, 5, 6, 6, 7, 8 is $\frac{9}{2}$. ■

The median of an ordered collection of numbers roughly partitions the collection into two halves: those below the median and those above. The **first quartile** is the median of the lower half. The **second quartile** is the median of the entire collection. The **third quartile** is the median of the upper half. Quartiles partition an ordered collection into four **quarters.**

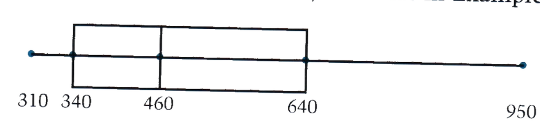
Example 3 *Finding the Quartiles*

The 15 golf winnings in Example 1 can be partitioned into quarters by the quartiles 340 (1st), 460 (2nd), and 640 (3rd).

1st Quarter:	310, 320, 340,	$x \le 340$
2nd Quarter:	420, 430, 450,	$340 \le x \le 460$
3rd Quarter:	490, 630, 640,	$460 \le x \le 640$
4th Quarter:	670, 680, 950	$640 \le x$

■

A **box-and-whisker plot** can be used to present a graphic summary of information shown, like that in Example 3.



What you should learn:

Goal How to use properties of algebra to prove a statement true and to use counter-examples to prove a statement false

Why you should learn it:

You will use the properties of algebra more formally in higher-level mathematics courses.

GOAL Using Properties of Algebra

Mathematics evolved over the past few thousand years in many stages. *All* of the early stages were centered around the use of mathematics to answer questions about real life. This is much like the way we wrote *Algebra 1*. We centered the concepts around the real-life use of mathematics.

As mathematical development matured, people began to collect and categorize the different rules, formulas, and properties that had been discovered. This took place independently in many different parts of the world: Africa, Asia, Europe, North America, and South America. The mathematics that we use today is a combination of the work of literally thousands of people.

As the collections and catalogs of rules, formulas, and properties grew, some discrepancies were noticed—some of the rules didn't seem to fit with some of the other rules. This made people wonder which rules were "true," and they tried to see which could be **proved.** After some effort, however, they realized that you can't prove *every* rule—some have to be accepted "on faith." In mathematics, the rules that we accept to be true *without proof* are called **postulates** or **axioms.** Many of the rules in Chapter 2 fall into this category.

The Basic Axioms of Algebra

Let a, b, and c be any real numbers.

Name of Axiom	Addition Axioms	Multiplication Axioms
Closure	$a + b$ is a real number.	ab is a real number.
Commutative	$a + b = b + a$	$ab = ba$
Associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Identity	$a + 0 = a, 0 + a = a$	$a(1) = a, 1(a) = a$
Inverse	$a + (-a) = 0$	$a\left(\frac{1}{a}\right) = 1, a \neq 0$
Axiom Relating Addition and Multiplication		
Distributive	$a(b + c) = ab + ac$	$(a + b)c = ac + bc$

Excerpts from Larson, Roland (1998). *Algebra: An Integrated Approach*. Evanston, IL. McDougal LIttell.

Once a list of axioms has been accepted, you can add more rules, formulas, and properties to the collection. Some of the new concepts are **definitions.** You don't have to prove a definition, but you do need to check that it is consistent with previous definitions and axioms.

Other new concepts, called **theorems,** do have to be proved. For instance, it is possible (although it is not done here) to use the basic axioms to prove that $c(-b) = -cb$ for all real numbers b and c.

Example 1 *Proving a Theorem*

Use the definition of subtraction, $a - b = a + (-b)$ and prove that the following theorem is true for all real numbers a, b, and c.

$$c(a - b) = ca - cb$$

Solution

$$\begin{aligned} c(a - b) &= c(a + (-b)) && \textit{Definition of subtraction} \\ &= ca + c(-b) && \textit{Distributive Property} \\ &= ca + (-cb) && \textit{Theorem listed above} \\ &= ca - cb && \textit{Definition of subtraction} \end{aligned}$$

Proofs are not easy to write or to read. They take a lot of practice. Our reason for listing this proof is simply to point out that when you are proving a new formula, *every* step must be justified by an axiom, a definition, or a previously proved theorem. ■

Sometimes people propose new rules that *are not* consistent with the axioms, definitions, and theorems in the accepted list. To show that a proposed rule is *inconsistent* or *false*, you need to find only one counterexample.

Example 2 *Finding a Counterexample*

Assign values to a and b to show that the following "rule" is inconsistent: $a + (-b) = (-a) + b$ for all real numbers a and b.

Solution Almost any choices of a and b will show the inconsistency. For instance, let $a = 1$ and let $b = 2$. Then

$$a + (-b) = 1 + (-2) = -1 \quad \text{and} \quad (-a) + b = -1 + 2 = 1.$$

Because -1 and 1 are not the same number, you have shown one case in which $a + (-b)$ is not equal to $(-a) + b$. Therefore, the proposed rule must be false. (If it were true, it would have to work for any choices of a and b.) ■