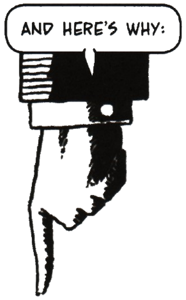
THE NUMBER LINE HOLDS ALL POSITIVE AND NEGATIVE WHOLE NUMBERS AND FRACTIONS. DO WE NEED ANY OTHER NUMBERS BESIDES THESE FOR MEASUREMENT? IN FACT, WE DO...



AS YOU MAY KNOW, YOU CAN TURN ANY FRACTION INTO A DECIMAL BY LONG DIVISION. HERE ARE 2/3, 5/8, AND 1/7:

```
     .6666...            .625
3 ) 2.0000...       8 ) 5.000
    1 8                 4 8
      20                  20
      18                  16
       20                  40
       18                  40
        20                  0
```

AND SO FORTH...

```
     .1428571428...
7 ) 1.0000000000
      7
      30
      28
       20
       14
        60
        56
         40
         35
          50
          49
           10
            7
            30
```

AND SO FORTH...

WHEN DIVIDING ONE WHOLE NUMBER BY ANOTHER, THERE ARE ONLY TWO POSSIBILITIES. THE DECIMAL EITHER

TERMINATES (ENDS, STOPS) AS WITH

5/8 = 0.625

OR **REPEATS A PATTERN** ENDLESSLY, LIKE

2/3 = 0.666666666.....

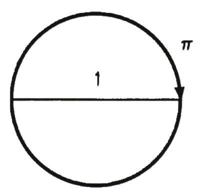
1/7 = 0.142857 142857 142857....

WHY? LOOK AT THE DIVISIONS ON THE LEFT. IF A REMAINDER IS EVER 0, THE DECIMAL TERMINATES. IF NOT, WELL... EACH REMAINDER MUST BE LESS THAN THE DIVISOR, SO THERE CAN BE ONLY SO MANY POSSIBLE REMAINDERS. AS YOU KEEP DIVIDING, YOU MUST HIT ONE OF THEM A SECOND TIME, AT WHICH POINT THE ENTIRE PATTERN MUST REPEAT.

IT SO HAPPENS THAT SOME NUMBERS HAVE AN EXPANSION THAT DOES **NOT** ENDLESSLY REPEAT A PATTERN. ONE EXAMPLE IS $\sqrt{2}$, THE SQUARE ROOT OF 2. (THIS IS THE NUMBER WHOSE PRODUCT WITH ITSELF IS **2**. MORE ON THIS LATER!)

$\sqrt{2}$ = 1.41421 35623 73095 04880....

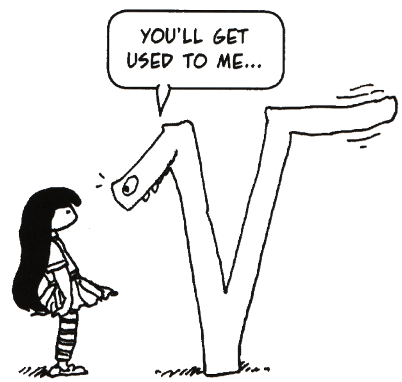
ANOTHER NON-REPEATER IS π, PI, THE DISTANCE AROUND A CIRCLE WITH DIAMETER = 1.



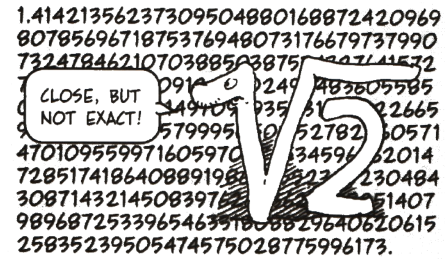
π = 3.14159 26535 89793 23846...

THESE NON-REPEATERS ARE CALLED **IRRATIONAL** NUMBERS, AND THEY HAVE THEIR PLACE ON THE NUMBER LINE, TOO.

BY THE WAY, "IRRATIONAL" DOESN'T MEAN WACKY OR UNPREDICTABLE, ALTHOUGH SOMETIMES IT MUST HAVE SEEMED THAT WAY. AT ONE TIME, SQUARE ROOTS USED TO BE CALLED "SURDS," AS IN **ABSURD**.



WHAT IRRATIONAL DOES MEAN IS THAT THESE NUMBERS CAN NEVER BE WRITTEN AS A **RATIO** OF TWO WHOLE NUMBERS—IN OTHER WORDS, AS A FRACTION. (A FRACTION'S DECIMAL EXPANSION MUST TERMINATE OR REPEAT.)



EVERY MEASURING NUMBER, THEN, IS ONE OF THESE:

Integer

A WHOLE NUMBER, POSITIVE OR NEGATIVE

Rational

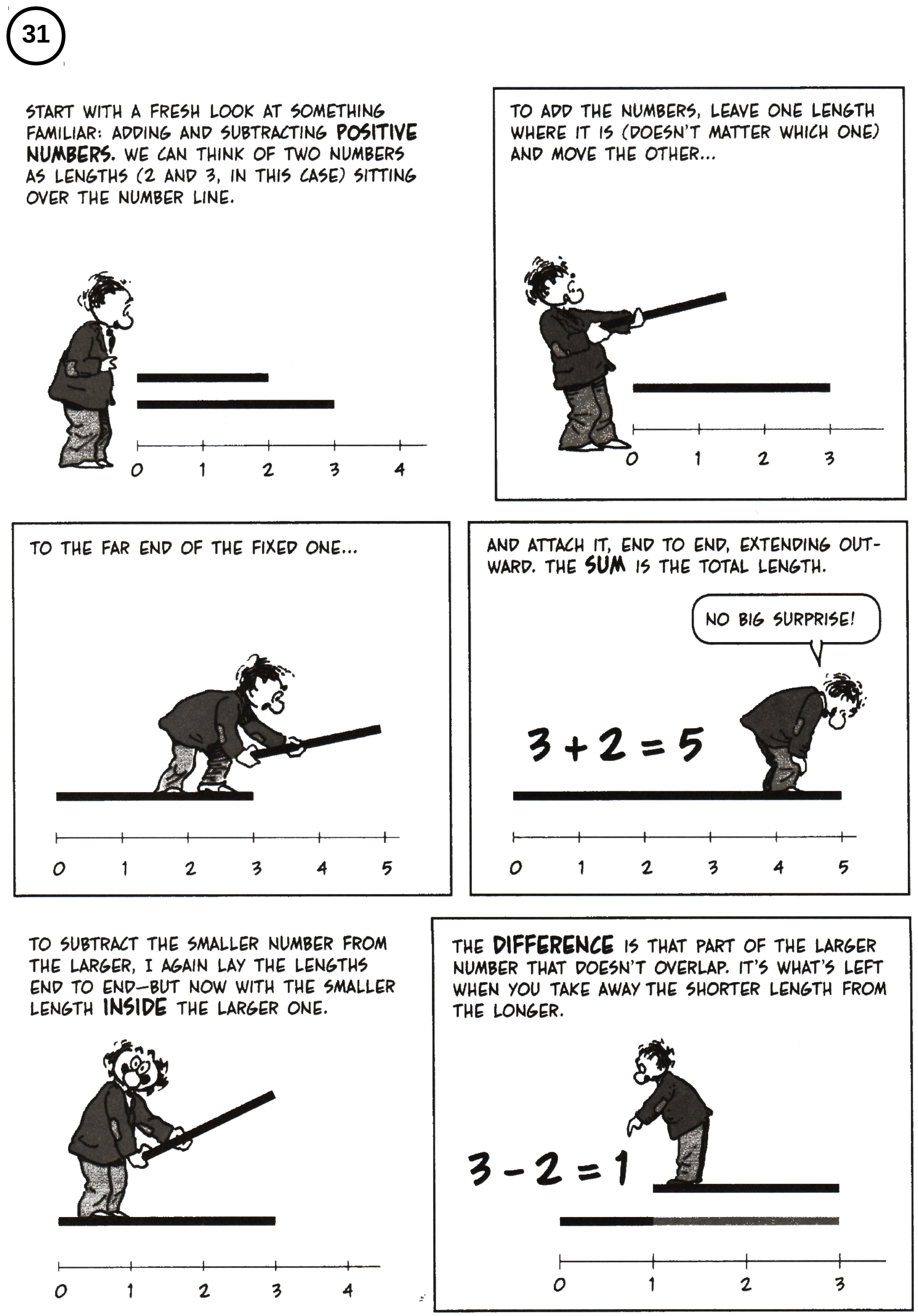
A NUMBER THAT **CAN** BE WRITTEN AS A FRACTION

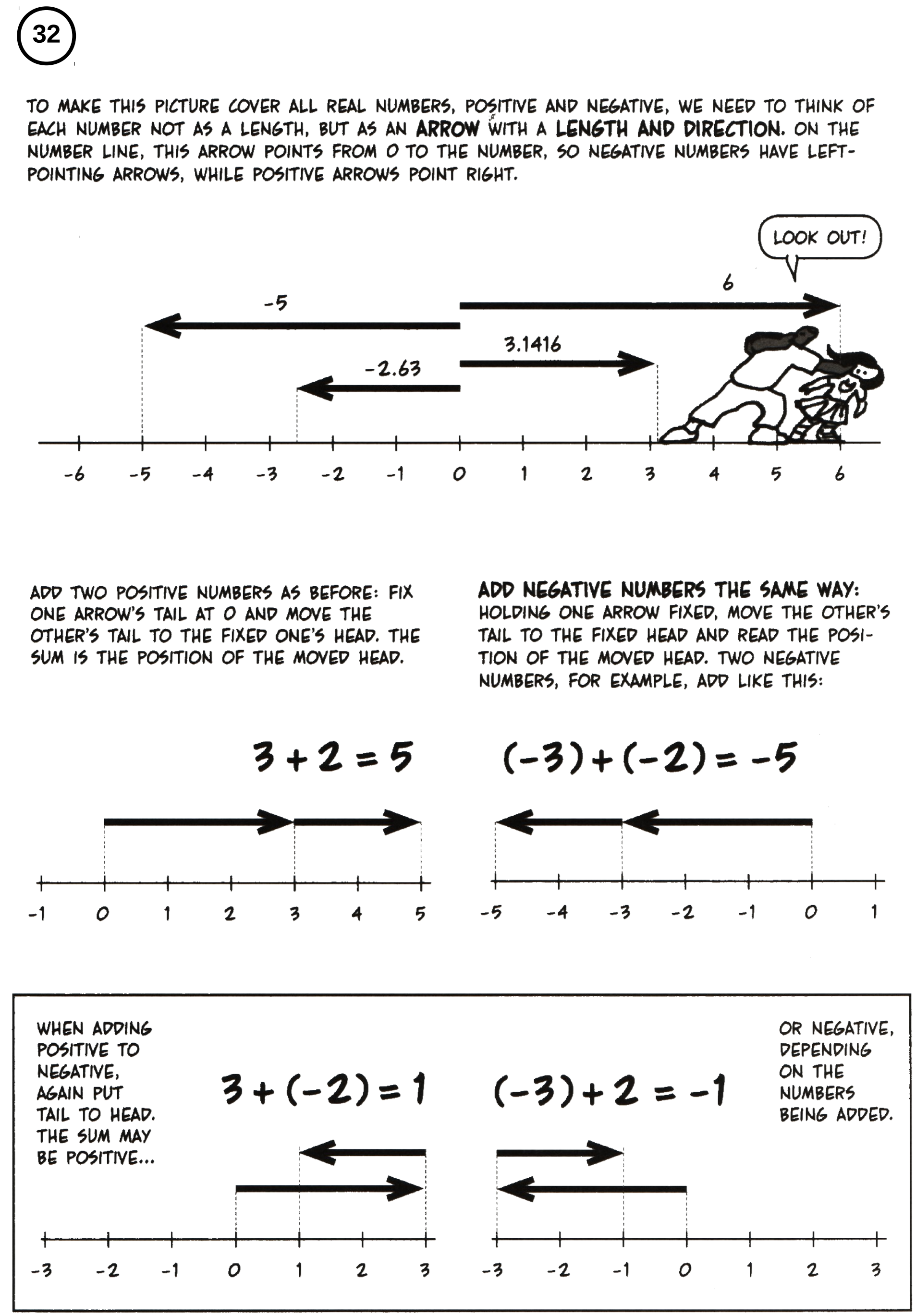
Irrational

ANYTHING ELSE

ALTOGETHER, THIS LINE FULL OF NUMBERS IS CALLED THE "REAL" NUMBERS, BUT WHETHER THEY'RE AS REAL AS, SAY, A ROCK OR A PIECE OF CHEESE I LEAVE FOR YOU TO DECIDE...

Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.





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