

AND HERE'S WHY:

A simple line drawing of a hand with the index finger pointing downwards. The hand is positioned below the 'AND HERE'S WHY:' text, indicating the start of the explanation.

AS YOU MAY KNOW, YOU CAN TURN ANY FRACTION INTO A DECIMAL BY LONG DIVISION. HERE ARE $\frac{2}{3}$, $\frac{5}{8}$, AND $\frac{1}{7}$:

$$\begin{array}{r} .6666\ldots \\ 3 \overline{) 2.0000\ldots} \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \end{array}$$

AND SO FORTH...

$$\begin{array}{r} .625 \\ 8 \overline{) 5.000} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$$\begin{array}{r} .1428571428... \\ 7 \overline{) 1.0000000000} \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \end{array}$$

AND SO
FORTH...

WHEN DIVIDING ONE WHOLE NUMBER BY ANOTHER, THERE ARE ONLY TWO POSSIBILITIES. THE DECIMAL EITHER

TERMINATES (ENDS, STOPS) AS WITH

$$5/8 = 0.625$$

OR REPEATS A PATTERN ENDLESSLY, LIKE

$$2/3 = 0.6666666666\dots$$

$$1/7 = 0.142857\ 142857\ 142857....$$

WHY? LOOK AT THE DIVISIONS ON THE LEFT. IF A REMAINDER IS EVER 0, THE DECIMAL TERMINATES. IF NOT, WELL... EACH REMAINDER MUST BE LESS THAN THE DIVISOR, SO THERE CAN BE ONLY SO MANY POSSIBLE REMAINDERS. AS YOU KEEP DIVIDING, YOU MUST HIT ONE OF THEM A SECOND TIME, AT WHICH POINT THE ENTIRE PATTERN MUST REPEAT.

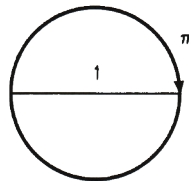
AS FAR AS THE
EYE CAN SEE,
AND FARTHER!

$$\frac{1}{11} = 0.0909090909$$

IT SO HAPPENS THAT SOME NUMBERS HAVE AN EXPANSION THAT DOES **NOT** ENDLESSLY REPEAT A PATTERN. ONE EXAMPLE IS $\sqrt{2}$, THE SQUARE ROOT OF 2. (THIS IS THE NUMBER WHOSE PRODUCT WITH ITSELF IS 2. MORE ON THIS LATER!)

$$\sqrt{2} = 1.41421\ 35623\ 73095\ 04880\dots$$

ANOTHER
NON-REPEATER
IS π , PI, THE
DISTANCE
AROUND A
CIRCLE WITH
DIAMETER = 1.



$$\pi = 3.14159\ 26535\ 89793\ 23846\dots$$

GETTING CROWDED
DOWN THERE...



BY THE WAY, "IRRATIONAL" DOESN'T MEAN WACKY OR UNPREDICTABLE, ALTHOUGH SOMETIMES IT MUST HAVE SEEMED THAT WAY. AT ONE TIME, SQUARE ROOTS USED TO BE CALLED "SURDS," AS IN **ABSRD**.

YOU'LL GET
USED TO ME...



WHAT IRRATIONAL DOES MEAN IS THAT THESE NUMBERS CAN NEVER BE WRITTEN AS A **RATIO** OF TWO WHOLE NUMBERS—IN OTHER WORDS, AS A FRACTION. (A FRACTION'S DECIMAL EXPANSION MUST TERMINATE OR REPEAT.)

1.14121356237309504880168872420969
8078569671875376948073176679737990
732478462107038853875 937143532
CLOSE, BUT
NOT EXACT!
09100249483505585
3494009355122665
5799950527820571
4701095599716059703345962014
7285174186408891983923230484
308714321450839763511407
98968725339654639108829640620615
25835239505474575028775996173.

EVERY MEASURING NUMBER, THEN, IS ONE OF THESE:

Integer

A WHOLE NUMBER,
POSITIVE OR NEGATIVE

Rational

A NUMBER THAT CAN BE WRITTEN AS A FRACTION

Irrational

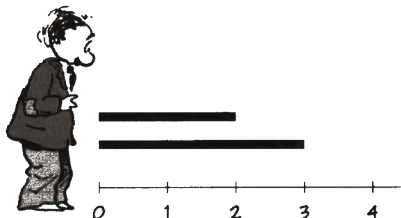
ANYTHING ELSE

ALTOGETHER,
THIS LINE FULL
OF NUMBERS
IS CALLED THE
"REAL" NUMBERS,
BUT WHETHER
THEY'RE AS REAL
AS, SAY, A ROCK
OR A PIECE OF
CHEESE I LEAVE
FOR YOU TO
DECIDE...

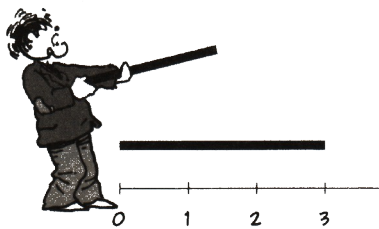
THEY GIVE ME A
HEADACHE, SO THEY
MUST BE REAL!



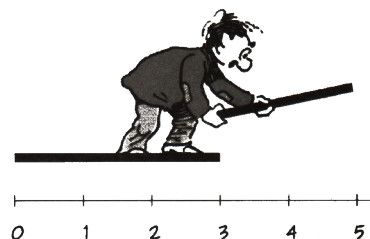
START WITH A FRESH LOOK AT SOMETHING FAMILIAR: ADDING AND SUBTRACTING **POSITIVE NUMBERS**. WE CAN THINK OF TWO NUMBERS AS LENGTHS (2 AND 3, IN THIS CASE) SITTING OVER THE NUMBER LINE.



TO ADD THE NUMBERS, LEAVE ONE LENGTH WHERE IT IS (DOESN'T MATTER WHICH ONE) AND MOVE THE OTHER...

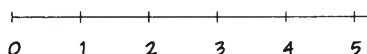


TO THE FAR END OF THE FIXED ONE...



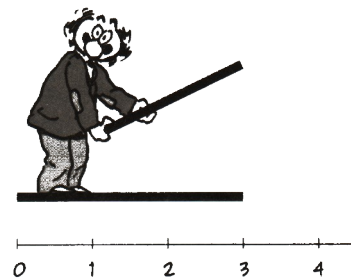
AND ATTACH IT, END TO END, EXTENDING OUTWARD. THE **SUM** IS THE TOTAL LENGTH.

$$3 + 2 = 5$$



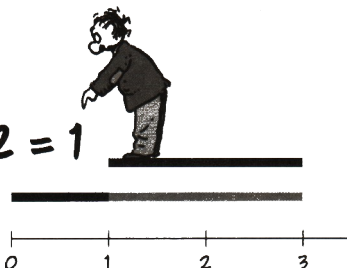
NO BIG SURPRISE!

TO SUBTRACT THE SMALLER NUMBER FROM THE LARGER, I AGAIN LAY THE LENGTHS END TO END—BUT NOW WITH THE SMALLER LENGTH **INSIDE** THE LARGER ONE.

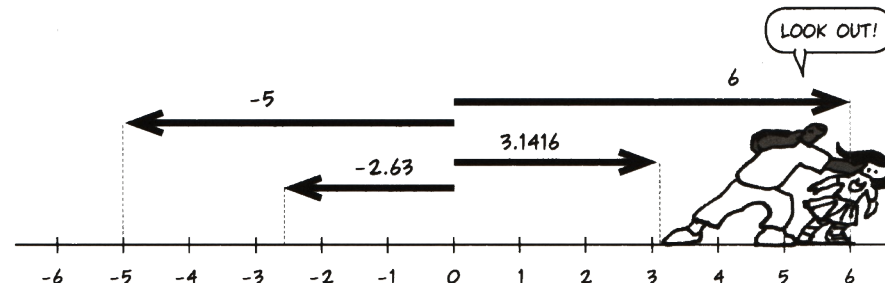


THE **DIFFERENCE** IS THAT PART OF THE LARGER NUMBER THAT DOESN'T OVERLAP. IT'S WHAT'S LEFT WHEN YOU TAKE AWAY THE SHORTER LENGTH FROM THE LONGER.

$$3 - 2 = 1$$

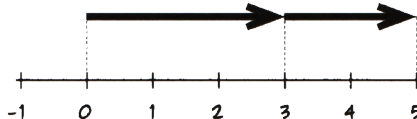


TO MAKE THIS PICTURE COVER ALL REAL NUMBERS, POSITIVE AND NEGATIVE, WE NEED TO THINK OF EACH NUMBER NOT AS A LENGTH, BUT AS AN **ARROW** WITH A **LENGTH AND DIRECTION**. ON THE NUMBER LINE, THIS ARROW POINTS FROM 0 TO THE NUMBER, SO NEGATIVE NUMBERS HAVE LEFT-POINTING ARROWS, WHILE POSITIVE ARROWS POINT RIGHT.



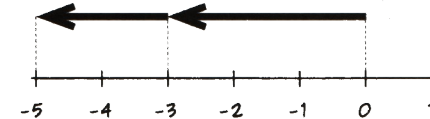
ADD TWO POSITIVE NUMBERS AS BEFORE: FIX ONE ARROW'S TAIL AT 0 AND MOVE THE OTHER'S TAIL TO THE FIXED ONE'S HEAD. THE **SUM** IS THE POSITION OF THE MOVED HEAD.

$$3 + 2 = 5$$



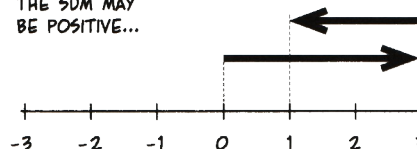
ADD **NEGATIVE NUMBERS** THE SAME WAY: HOLDING ONE ARROW FIXED, MOVE THE OTHER'S TAIL TO THE FIXED HEAD AND READ THE POSITION OF THE MOVED HEAD. TWO **NEGATIVE** NUMBERS, FOR EXAMPLE, ADD LIKE THIS:

$$(-3) + (-2) = -5$$

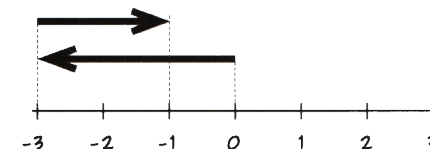


WHEN ADDING POSITIVE TO NEGATIVE, AGAIN PUT TAIL TO HEAD. THE **SUM** MAY BE POSITIVE...

$$3 + (-2) = 1$$



$$(-3) + 2 = -1$$



OR NEGATIVE, DEPENDING ON THE NUMBERS BEING ADDED.