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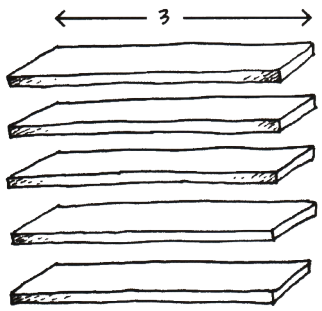
INSTEAD OF CUTTING UP BODIES, LET'S START BY BUILDING A BOOKCASE. IT WILL HAVE 5 SHELVES, AND EACH SHELF WILL BE 3 FEET LONG. THE SHELVES' TOTAL LENGTH IS OBVIOUSLY

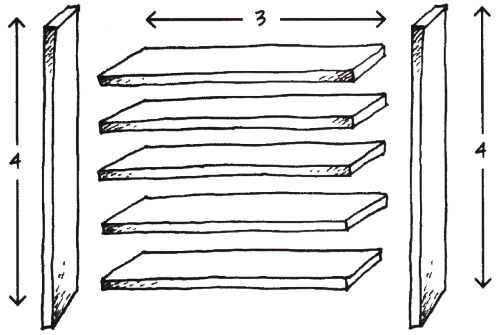
5×3 FEET

(I KNOW, I KNOW, THAT'S 15 FEET, BUT WE DON'T CARE ABOUT THAT AT THE MOMENT...)



IF WE ADD TWO 4-FOOT SIDES, THE AMOUNT OF LUMBER INCREASES... AND WE HAVE TO ADD THIS MUCH:

2×4 FEET



SO THIS **NUMERICAL EXPRESSION,** THE SUM OF THE TWO PRODUCTS, GIVES THE TOTAL LENGTH OF ALL THE BOARDS:

$$(5 \times 3) + (2 \times 4)$$

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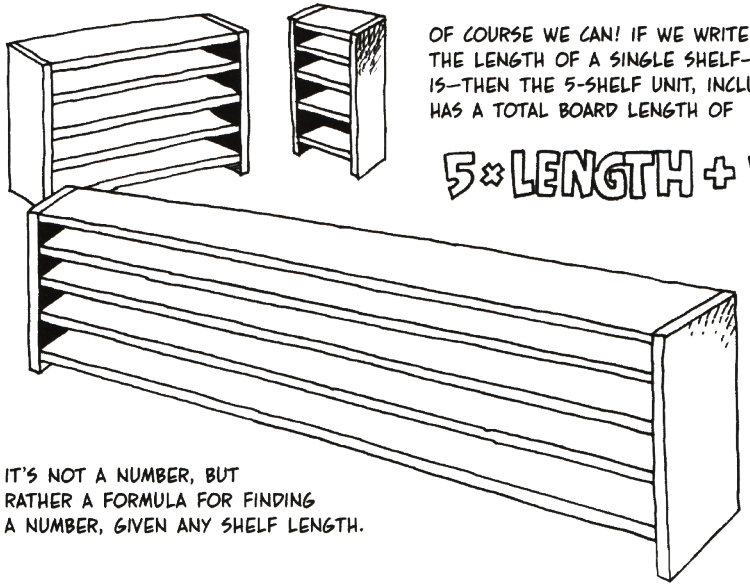
NOW LET'S GET

ALGEBRAIC!

THIS PAGE, READER, MARKS THE PLACE WHERE WE CROSS FROM THE OLD, FAMILIAR GROUND OF ARITHMETIC TO THE PROMISED LAND OF ALGEBRA.

THE CHANGE BEGINS WITH A QUESTION ABOUT OUR BOOKCASE: CAN WE WRITE AN EXPRESSION FOR THE TOTAL LENGTH OF ALL BOARDS OF A 4-FOOT-TALL BOOKCASE WITH 5 SHELVES **OF ANY LENGTH?**



OF COURSE WE CAN! IF WE WRITE "LENGTH" FOR THE LENGTH OF A SINGLE SHELF—WHATEVER IT IS—THEN THE 5-SHELF UNIT, INCLUDING ITS SIDES, HAS A TOTAL BOARD LENGTH OF

$5 \times \text{LENGTH} + 2 \times 4$

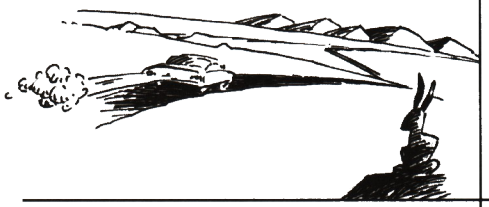
IT'S NOT A NUMBER, BUT RATHER A FORMULA FOR FINDING A NUMBER, GIVEN ANY SHELF LENGTH.

Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.

More Examples

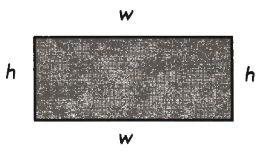
IF YOU MOVE AT A STEADY SPEED OF 60 MILES PER HOUR, THEN IN t HOURS YOU TRAVEL A DISTANCE OF

$60t$ MILES



A RECTANGLE'S **AREA** IS THE PRODUCT OF THE HEIGHT h AND ITS WIDTH w. AREA = hw.

ITS **PERIMETER** IS THE TOTAL DISTANCE AROUND THE RECTANGLE, THE SUM OF ALL ITS SIDES. YOU COULD WRITE THIS AS EITHER

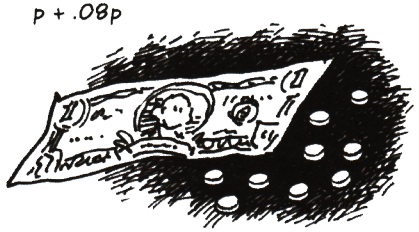


$2h + 2w$ (DOUBLE EACH SIDE, THEN ADD) OR

$2(h + w)$ (ADD HEIGHT TO WIDTH, THEN DOUBLE)

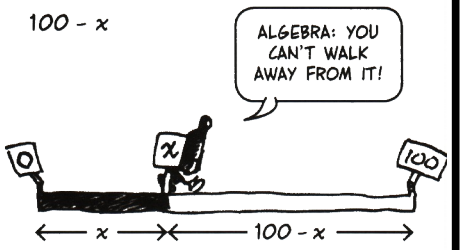
WHERE I LIVE, **THE SALES TAX RATE** IS 8% (THAT'S 8/100 = .08). IF AN ITEM IS MARKED WITH A PRICE p, THEN THE SALES TAX IS $.08p$. THE PRICE I ACTUALLY PAY IS THE MARKED PRICE PLUS THE TAX, OR

$p + .08p$



IF YOU PLAN TO WALK 100 MILES, AND YOU'VE ALREADY COVERED x OF THEM, THEN THE DISTANCE STILL TO GO IS

$100 - x$

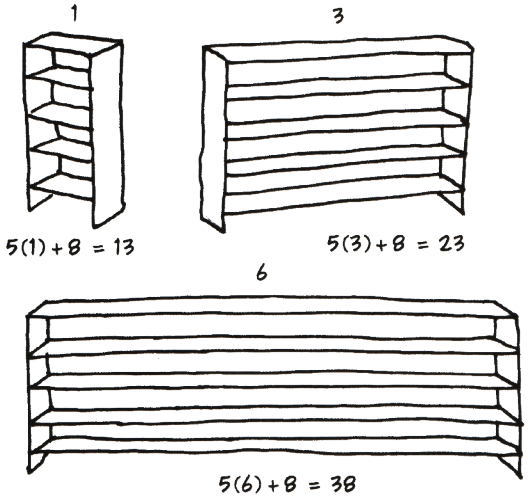


YOU READ ALGEBRAIC EXPRESSIONS MUCH AS YOU READ TEXT MESSAGES: ONE LETTER, NUMBER, OR SYMBOL AT A TIME—EXCEPT THAT PARENTHESES INDICATE GROUPING. WHATEVER IS INSIDE PARENTHESES IS CALLED A "QUANTITY."

EXPRESSION	HOW TO SAY IT	MEANING
$a + x$	"AY PLUS EX"	THE SUM OF TWO NUMBERS
$5y$	"FIVE WYE"	FIVE TIMES A NUMBER
$\frac{x}{2}$	"EX OVER TWO"	HALF A NUMBER
$-a$	"NEGATIVE AY"	THE NEGATIVE OF A NUMBER
$5T + 1$	"FIVE TEE PLUS ONE"	ONE MORE THAN FIVE TIMES A NUMBER
$5(x + 1)$	"FIVE TIMES THE QUANTITY EX PLUS ONE"	FIVE TIMES THE SUM OF A NUMBER AND ONE

EVALUATING EXPRESSIONS

UNLIKE A NUMERIC EXPRESSION, AN ALGEBRAIC EXPRESSION HAS NO DEFINITE VALUE: $5L + 8$ ISN'T A NUMBER. INSTEAD, IT'S A SORT OF RECIPE DESCRIBING EXACTLY HOW TO CALCULATE A NUMBER FOR EACH VALUE OF L.



TO FIND THE VALUE OF $5L+8$ FOR SOME VALUE OF L, YOU SUBSTITUTE (OR "PLUG IN") THAT NUMBER FOR L AND THEN DO THE ARITHMETIC AS SHOWN HERE. THIS IS CALLED **EVALUATING THE EXPRESSION** FOR A PARTICULAR **VALUE OF THE VARIABLE.**

Evaluation Example 1.

EVALUATE $p + .08p$ WHEN $p = 50$.

STEP 1. PLUG IN 50 WHEREVER YOU SEE p TO GET THE NUMERICAL EXPRESSION

$50 + (.08)(50)$

STEP 2. DO THE ARITHMETIC.

$50 + (.08)(50) = 50 + 4$

$= 54$

WE CAN ALSO EVALUATE EXPRESSIONS OF MORE THAN ONE VARIABLE, GIVEN VALUES FOR THOSE VARIABLES.

Evaluation Example 2. EVALUATE

$2(h + w)$ WHEN $h = 3$ AND $w = 7$.

STEP 1. PLUG IN THE VALUES TO GET

$2(3 + 7)$

STEP 2. DO THE ARITHMETIC.

$2(3 + 7) = 2 \times 10 = 20$

Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.