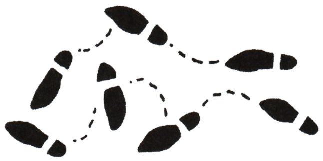
1

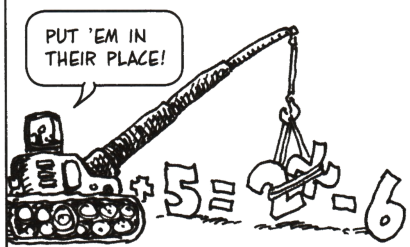
HOW TO SOLVE AN EQUATION, STEP BY STEP

(SOME EQUATIONS, ANYWAY)

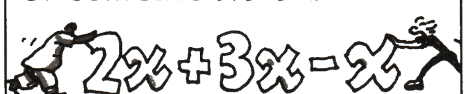


1. "Prep" THE EQUATION IF NECESSARY BY GETTING RID OF PARENTHESES AND COMBINING LIKE TERMS. ("LIKE" MEANS THAT CONSTANTS ADD WITH CONSTANTS, VARIABLE TERMS WITH VARIABLE TERMS.)

2. Isolate, BY ADDITION AND/OR SUBTRACTION, THE CONSTANTS ON ONE SIDE (USUALLY THE RIGHT) AND THE VARIABLE TERMS ON THE OTHER (USUALLY THE LEFT).



3. Combine LIKE TERMS.



THE EQUATION WILL NOW LOOK LIKE THIS: (SOME NUMBER)x = SOME OTHER NUMBER.

4. Multiply BOTH SIDES BY THE **RECIPROCAL** OF THE NUMBER IN FRONT OF THE VARIABLE. THIS NUMBER IS CALLED THE VARIABLE'S **COEFFICIENT.** FOR INSTANCE, GIVEN

$$4x = 12$$

4 IS THE COEFFICIENT OF x. MULTIPLYING BY $\frac{1}{4}$ GIVES

$$x = 3$$

AND THE EQUATION IS SOLVED.

5. Check THE ANSWER. THIS IS IMPORTANT, BOTH TO CHECK YOUR WORK AND FOR ANOTHER REASON TO BE EXPLAINED SHORTLY.

2

HERE'S A COMPLICATED EQUATION THAT NEEDS SOME PREP WORK TO SOLVE.

Example 2.

$$2(x-1)+3(x-2)+x = 2x+4$$

WE GO STEP BY STEP.

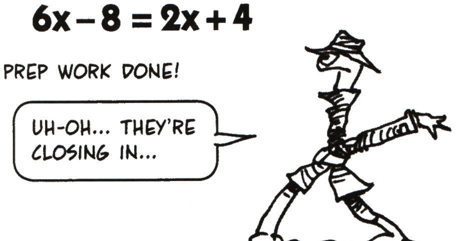
1. THOSE PARENTHESES MAKE IT HARD TO SEE WHAT TO CLEAR FROM EACH SIDE, SO LET'S GET RID OF THEM. BY THE DISTRIBUTIVE LAW, $2(x-1) = 2x-2$ AND $3(x-2) = 3x-6$, MAKING THE EQUATION

$$2x-2+3x-6+x = 2x+4$$

COMBINING LIKE TERMS, VARIABLE WITH VARIABLE, CONSTANT WITH CONSTANT, GIVES:

$$6x-8 = 2x+4$$

PREP WORK DONE!



2. NOW REBALANCING IS EASY: SUBTRACTING **2x** CLEARS THE VARIABLE TERM FROM THE RIGHT, AND ADDING **8** CLEARS THE CONSTANT FROM THE LEFT.

$$\begin{array}{rcr} 6x-8 & = & 2x+4 \\ -2x+8 & & -2x+8 \\ \hline 6x-2x & = & 4+8 \end{array}$$

3. COMBINE TERMS: $6x-2x = 4x$ AND $4+8 = 12$. NOW THE EQUATION IS

$$4x = 12$$

4. DIVIDING BOTH SIDES BY 4, THE COEFFICIENT OF x, SOLVES IT.

$$x = 3$$

5. FINALLY, CHECK THE ANSWER BY PLUGGING IN 3 FOR x THE ORIGINAL EQUATION:

$$2(3-1)+3(3-2)+3 \stackrel{?}{=} 2\cdot3+4$$
$$2\cdot2+3\cdot1+3 \stackrel{?}{=} 6+4$$
$$4+3+3 \stackrel{?}{=} 6+4$$
$$10 = 10$$

Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.

NEGATIVE COEFFICIENTS

AFTER REBALANCING, YOU MAY FIND A NEGATIVE COEFFICIENT STUCK TO THE UNKNOWN, LIKE THIS:

$$-3x = -9$$

AT THIS POINT, YOU COULD DIVIDE BY −3 (OKAY BUT MESSY), BUT IT'S JUST A LITTLE EASIER TO MULTIPLY BOTH SIDES BY −1 AND MAKE THE COEFFICIENT POSITIVE.

$$3x = 9$$

IN A SIMILAR WAY, ADDING **FRACTIONAL TERMS** CAN BE SLOW AND ANNOYING. LUCKILY, YOU CAN CLEAR ALL FRACTIONS FROM AN EQUATION BY MULTIPLYING THROUGH BY A COMMON DENOMINATOR. GIVEN AN EQUATION LIKE THIS ONE:

$$\frac{3}{2}x + \frac{1}{3} = \frac{5}{6}x + 2$$

MULTIPLY BOTH SIDES BY 6 (THE LEAST COMMON DENOMINATOR OF 2, 3, AND 6).

$$\frac{3 \times 6}{2}x + \frac{1 \times 6}{3} = \frac{5 \times 6}{6}x + (6)(2)$$

AFTER CANCELING COMMON FACTORS, ALL FRACTIONS ARE GONE!

$$9x + 2 = 5x + 12$$

THIS REBALANCES TO

$$4x = 10 \quad \text{AND SO}$$

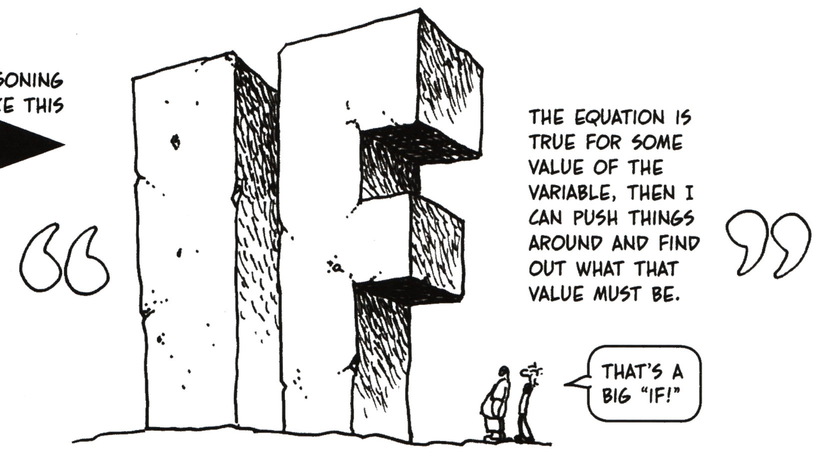
$$x = \frac{5}{2}$$

TRY CHECKING THE SOLUTION.

A WORD ABOUT CHECKING: IT'S IMPORTANT FOR AN OBVIOUS REASON: PEOPLE MAKE MISTAKES!

THERE'S ANOTHER REASON AS WELL. IT HAS TO DO WITH ALGEBRA'S BASIC THINKING, WHICH ASSUMES, WHEN REBALANCING, THAT THE EQUATION WAS **TRUE.**

THE REASONING GOES LIKE THIS



HOW ABOUT THIS EQUATION?

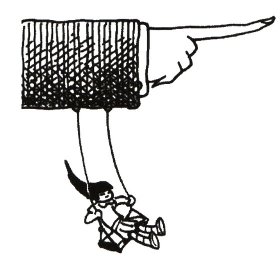
$$\mathbf{x = x + 1}$$

BLINDLY REBALANCING, WE CLEAR x FROM THE RIGHT

$$\begin{array}{rcl} x & = & x+1 \\ -x & & -x \\ \hline 0 & = & 1 \end{array}$$

AND CONCLUDE THAT $0 = 1$. OOPS!

THIS HAPPENED BECAUSE THE ORIGINAL EQUATION COULD NEVER HAVE BEEN TRUE IN THE FIRST PLACE. HOW CAN ANY NUMBER BE ONE MORE THAN ITSELF? THE EQUATION HAS **NO SOLUTION.**



CHECKING SOLUTIONS ASSURES US THAT THE ORIGINAL ASSUMPTION WAS OKAY: THE SOLVED EQUATION REALLY WAS TRUE FOR SOME VALUE OF THE VARIABLE.

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