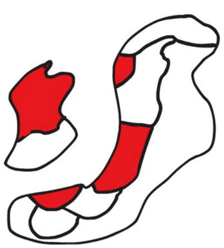
A deceptively simple mathematical problem lurks within the brightly colored maps showing the nations of Europe or the patchwork of states in the United States. It's the sort of problem that might worry frugal mapmakers who insist on decorating their maps with as few colors as possible. The question is whether four colors are always enough to fill in every conceivable map that can be drawn on a flat piece of paper so that no countries sharing a common boundary are the same color.

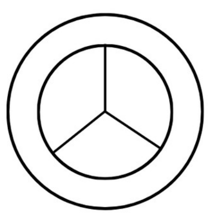
An additional definition and a condition turn this mapmakers' conundrum into a well-defined mathematical problem. A single shared point doesn't count as a shared border. Otherwise, a map whose countries are arranged like the wedges of a pie would need as many colors as there are countries. Moreover, countries must be connected regions; they can't have colonies scattered all over the map *(see Figure 1.1, top left and middle).*

First proposed in 1852 by British graduate student Francis Guthrie in a letter to his younger brother, the four-color problem has since intrigued and stumped professional and amateur mathematicians alike. Early on, mathematicians realized that three colors are certainly not enough for every possible map. It's easy to draw a map that needs four colors *(see Figure 1.1, top right).* English mathematician Augustus De Morgan also proved that it was impossible for five countries to be placed so that each one of them borders the other four. That led him to believe that five colors would never be needed. However, the proof that exactly four colors suffice remained elusive despite years of map sketching and attempted proofs.

Illegal map with disconnected region



Map that needs 4 colors



In 1976, two mathematics professors finally proved the four-color theorem. It was the kind of news that mathematicians greet with champagne toasts. When Kenneth Appel and Wolfgang Haken of the University of Illinois announced their success, there were great celebrations in mathematical circles. They had scaled one of the Mount Everests of mathematics.

But there was a shock awaiting anyone curious about how the four-color theorem had been proved. The proof was unlike any other mathematical proof that had ever been done. Its complex text—strewn with diagrams and filling hundreds of pages—daunted all but the most determined readers. Even more dismaying to many mathematicians was that for the first time, a computer had been used as a sophisticated accountant to verify certain facts needed for the proof.

The computer allowed Haken and Appel to analyze a large collection of possible cases, a collection shown by mathematical analysis to be sufficient to prove the theorem. That task took 1,200 hours on a fast computer; it would have taken practically forever by hand. That also meant the proof could not be verified without the aid of a computer. Furthermore, Appel and Haken had tried various strategies in computer experiments to perfect several ideas that were essential for the proof.

	Addition	Multiplication	Square Root
Computer	0.3 ns	14.93 ns	10 ns
Average human	2-3sec	2-3sec	1-2min

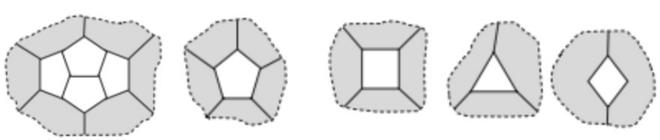
Excerpts from Peterson, I. (1988). *The Mathematical Tourist*. New York, NY. W. H. Freeman and Company.

Even now, doubts still linger about the validity of the proof of the four-color theorem. Although most mathematicians accept the idea that the Haken-Appel approach works, persistent rumors insist there is something wrong with the proof. Some people hint that the computer program was faulty; others, that the method itself is wrong in some way. A few complain that the computer program can't be verified properly. Nevertheless, the proof continues to hold up. Despite the discovery of such minor flaws as typographical and copying errors, the proof's basic strength has not been threatened.

To Appel and Haken, their proof of the four-color theorem represents a new and interesting facet of mathematics. Although a shorter, more elegant proof may someday be found, it's also possible that no such proof exists. The theorem may be one for which there never will be a proof short enough to be readily understood at a glance. It's an example of a curious occurrence in mathematics: the coupling of a simple, succinct problem with an incredibly complicated proof.

More than a decade after Haken and Appel's proof of the four-color theorem, computers are still scarce among mathematicians and seldom used for serious mathematical research. One mathematician, Stanford University's Joseph Keller, even remarked in 1986 that at his university, the mathematics department has fewer computers than any other department, including French literature. Many mathematicians have the feeling that using a computer is akin to cheating and say that computation is merely an excuse for not thinking harder.

The general mechanic of the four-color theorem is that all situations can be collapsed to one of or a combination of a set of solvable scenarios such as this:



Still, computers are beginning to creep into mathematics. By providing vivid images that suggest new questions, they are helping to mend a rift that had developed between pure and applied mathematics. They are linking mathematical questions in computer science with computational questions in mathematics. These changes are enriching a subject that many outsiders have regarded as an abstract, even useless, pursuit.

The availability of very fast computers with large memories has led to two important developments in science and technology, both of which rely on mathematics. The first development is the use of mathematical equations to represent a physical situation, such as the air rushing over an airplane's wing or the chemical reactions leading to the formation of acid rain. When they are good enough, such mathematical models allow researchers to replace many of their wind-tunnel and test-tube experiments with computer manipulations.

The second development arises because computers are invaluable for extracting relationships and patterns hidden deep within vast amounts of data. For example, they can pinpoint the location of an oil field using seismic data or the site of a brain tumor using x-ray absorption measurements. The storage and processing of data require subtle, sophisticated techniques. More often than not, a piece of abstract mathematics worked out years before—and believed to be totally without practical value—finds a role in the "real" world.

The Bailey–Borwein–Plouffe formula was discovered using computer aided research, or experimental mathematics. It is the first formula to discover a way to calculate the nth digit of π without calculating n digits before, and does so for base 16 digits.

$$\pi = \sum_{k=0}^{\infty} \left[\frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right) \right]$$

This formula can be modified to calculate the nth digit of other irrational numbers and to calculate the nth decimal digit of π, though no system to do this can be found and all current formulas based on the Bailey–Borwein–Plouffe were found through trial and error.

$$\pi = \sum_{k=0}^{\infty} \left[\frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right) \right]$$

Excerpts from Peterson, I. (1988). *The Mathematical Tourist*. New York, NY. W. H. Freeman and Company.