What you should learn:

Goal 1 How to use properties of perpendicular bisectors

Goal 2 How to use properties of angle bisectors

Why you should learn it:

Perpendicular bisectors and angle bisectors can be used to solve real-life problems such as in navigation or plumbing.

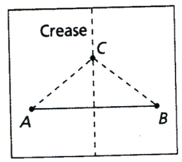
Goal 1 Using Perpendicular Bisectors

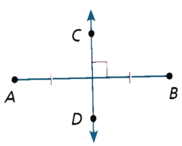
In the Chapter Exploration, you were asked to find locations that were the same taxi distance from two given locations. The following investigation explores locations that are the same straight-line distance from two given points.

LESSON INVESTIGATION

Investigating Distances

Draw a segment, $\overline{AB}$, on a piece of paper. Fold the paper so that A and B touch. Choose a point, C, on the crease line that is not the midpoint of $\overline{AB}$, as shown at the left. Compare the lengths of $\overline{AC}$ and $\overline{BC}$.

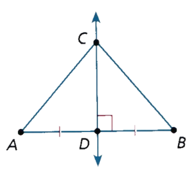




In the figure at the left, the line $\overleftrightarrow{CD}$ is the **perpendicular bisector** of $\overline{AB}$ because it is perpendicular to $\overline{AB}$, and it bisects $\overline{AB}$.

Theorem 5.1 Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.



Proof: Refer to the figure at the left.

Given: $\overleftrightarrow{CD} \perp \overline{AB}$, $\overline{AD} \cong \overline{DB}$ ***Prove:*** $\overline{AC} \cong \overline{BC}$

Statements	*Reasons*
1. $\overline{AD} \cong \overline{DB}$, $\overleftrightarrow{CD} \perp \overline{AB}$	**1.** Given
2. $\angle ADC$ and $\angle BDC$ are right angles.	**2.** 2 $\perp$ lines intersect to form right $\angle$s.
3. $\angle ADC \cong \angle BDC$	**3.** All right angles are congruent.
4. $\overline{CD} \cong \overline{CD}$	**4.** Reflexive Prop. of Congruence
5. $\triangle ADC \cong \triangle BDC$	**5.** SAS Congruence Postulate
6. $\overline{AC} \cong \overline{BC}$	**6.** CPCTC

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Excerpts from Larson, Roland (1998). *Geometry: An Integrated Approach*. Evanston, IL. McDougal LIttell.

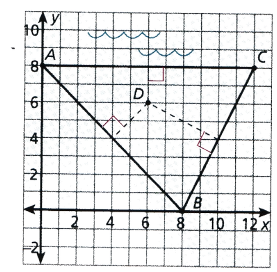
Theorem 5.2 Perpendicular Bisector Converse

If a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

Real Life **Navigation**

Example 1 *A Point Equidistant from Three Points*

Three sailboats are located at $A(0, 8)$, $B(8, 0)$, and $C(12, 8)$, as shown at the left. Is there a point that is equidistant from all three sailboats?



Solution All points that are equidistant from A and B lie on the perpendicular bisector of $\overline{AB}$. This bisector has a slope of 1 and passes through the midpoint, (4, 4), of $\overline{AB}$. Thus, you can write the equation of the perpendicular bisector of $\overline{AB}$ as

$y = x$. *Perpendicular bisector of* $\overline{AB}$

All points that are equidistant from A and C lie on the perpendicular bisector of $\overline{AC}$. This bisector is the vertical line whose equation is

$x = 6$. *Perpendicular bisector of* $\overline{AC}$

To find the point that is equidistant from A, B, and C, you can find the intersection of both of these perpendicular bisectors. Using substitution, you can determine that the point is

$D(6, 6)$. *D is equidistant from A, B, and C.*

You can use the Distance Formula to check that D is the same distance from A, B, and C.

Distance between A and D:

$$\sqrt{(0-6)^2 + (8-6)^2} = \sqrt{36+4} = \sqrt{40}$$

Distance between B and D:

$$\sqrt{(8-6)^2 + (0-6)^2} = \sqrt{4+36} = \sqrt{40}$$

Distance between C and D:

$$\sqrt{(12-6)^2 + (8-6)^2} = \sqrt{36+4} = \sqrt{40}$$

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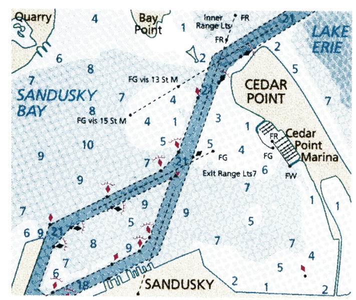

To find distances over water, sailors use charts such as the one shown above and radio direction finders (RDF's) that receive transmissions from fixed signals whose locations are known.

In Example 1, point D was the intersection of the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$. You could also have solved the problem by finding the intersection of the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$. Try doing that. You should obtain the same point of intersection.

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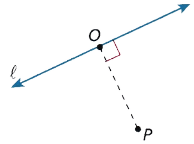
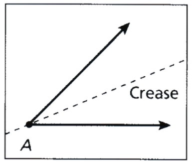
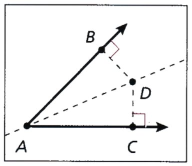
Goal 2 Using Angle Bisectors



The *distance* between a point and a line is the length of the perpendicular segment from the point to the line. For instance, in the diagram at the right, the distance between the point P and the line ℓ is PO.





LESSON INVESTIGATION

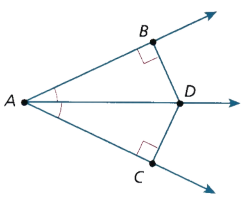
▪ *Investigating Distances*

Draw an angle, $\angle A$, on a piece of paper. Fold the paper so that the two sides of the angle lie on each other. The crease line is the bisector of $\angle A$. Draw a point D on the crease, as shown at the left. Measure the distance between D and each side of $\angle A$. What can you conclude?

From this investigation, it appears that any point on the angle bisector of $\angle A$ is the same distance from each side of the angle. This result is stated as Theorem 5.3.

Theorem 5.3 Angle Bisector Theorem If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

Theorem 5.4 Angle Bisector Converse If a point is in the interior of an angle and equidistant from the sides of an angle, then it lies on the bisector of the angle.



Proof of Theorem 5.3: Refer to the figure at the left.

Given: D is on the bisector of $\angle BAC$, $\overline{BD} \perp \overline{AB}$, $\overline{CD} \perp \overline{AC}$

Prove: $\overline{BD} \cong \overline{CD}$

Statements	*Reasons*
1. D is on bisector of $\angle BAC$	**1.** Given
2. $\angle BAD \cong \angle CAD$	**2.** Def. of angle bisector
3. $\overline{BD} \perp \overline{AB}$	**3.** Given
4. $\overline{CD} \perp \overline{AC}$	**4.** Given
5. $\angle ABD$ and $\angle ACD$ are right angles.	**5.** 2 ⊥ lines intersect to form four right ∠s.
6. $\angle ACD \cong \angle ABD$	**6.** All right angles are congruent.
7. $\overline{AD} \cong \overline{AD}$	**7.** Reflexive Prop. of Congruence
8. $\triangle ADB \cong \triangle ADC$	**8.** AAS Congruence Postulate
9. $\overline{BD} \cong \overline{CD}$	**9.** CPCTC ■

Excerpts from Larson, Roland (1998). *Geometry: An Integrated Approach*. Evanston, IL. McDougal LIttell.

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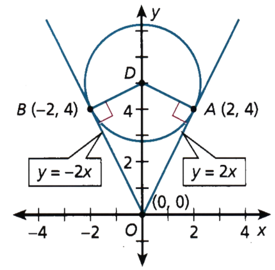
Real Life **Plumbing**

Example 2 *Finding Distances in a Coordinate Plane*

A pipe is supported by a V-shaped frame, as shown at the left. The sides of the frame are given by the equations $y = 2x$ and $y = -2x$. The pipe touches the sides of the frame at the points $A(2, 4)$ and $B(-2, 4)$. Find the distance between the center of the pipe and the vertex of the frame.



Solution From the diagram, you can see that the center of the pipe lies on the line $\overleftrightarrow{BD}$. This line has a slope of $\frac{1}{2}$ and passes through the point $B(-2, 4)$. You can use the slope-intercept form to find an equation of $\overleftrightarrow{BD}$.

$y = mx + b$	*Slope-intercept form*
$4 = \frac{1}{2}(-2) + b$	*Substitute 4 for y, $\frac{1}{2}$ for m, and −2 for x.*
$5 = b$	*Solve for b.*
$y = \frac{1}{2}x + 5.$	*Equation of $\overleftrightarrow{BD}$*

The center of the pipe is the intersection of $\overleftrightarrow{BD}$ and the y-axis. Because the y-intercept of the line $y = \frac{1}{2}x + 5$ is 5, it follows that the intersection of $\overleftrightarrow{BD}$ and the y-axis is

$D(0, 5).$ *Center of pipe*

Using these coordinates, you can determine that the center of the pipe is 5 units from the vertex of the frame. ■

Communicating about GEOMETRY

▶ SHARING IDEAS about the Lesson

Extending the Example

Problem Solving **Make a Model**

A. Use graph paper to draw the diagram of the pipe and frame shown above. Find the radius of the pipe by calculating the distance AD.

B. Show that $AD = BD$ by calculating the distance BD. Explain why D lies on the angle bisector of $\angle BOA$.

C. Use a protractor to measure $\angle AOD$ and $\angle DOB$. Are the measures the same?

D. Suppose you doubled the radius of the pipe. How far would the center of the larger pipe lie from the vertex of the frame? Answer the question by cutting a paper circle whose radius is $2(AD)$. Use it and your diagram as a model, and measure the distance from the center of the circle to the vertex of the frame.