What you should learn:

Goal 1 How to identify the three basic rigid transformations in a plane

Goal 2 How to use transformations to identify patterns and their properties in real life

Why you should learn it:

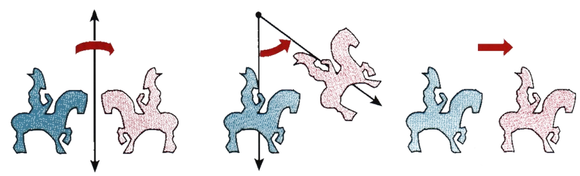
You can use transformations to create visual patterns, such as stencil patterns for the border of a wall.

Goal 1 Identifying Transformations

Figures in a plane can be reflected, rotated, or slid to produce new figures. The new figure is the **image,** and the original figure is the **preimage.** The operation that maps (or moves) the preimage onto the image is called a **transformation.**

In this chapter, you will learn about three basic transformations—reflections, rotations, and translations—and combinations of these. They are closely related to the concept of congruence.

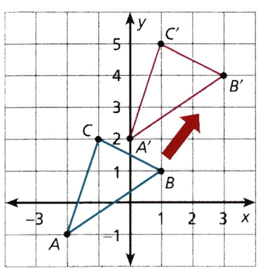
For each of the three transformations show below, the blue horseman is the preimage and the red horseman is the image. This color convention will be used throughout this text.



Reflection *Rotation* *Translation*

Example 1 *Identifying Transformations*

Identify the transformation shown at the left.



Solution This transformation is a translation. To obtain $\triangle A'B'C'$, each point of $\triangle ABC$ was slid 2 units to the right and 3 units up.

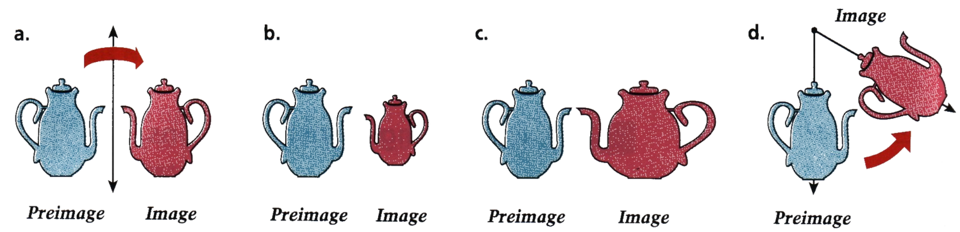
The two triangles are congruent. You can use the Distance Formula to show that each side of $\triangle ABC$ has the same measure as the corresponding side of $\triangle A'B'C'$. ■

In Example 1, note that the translation maps every point of the plane onto another point of the plane. In doing this, the translation maps every figure onto a congruent figure. Such a transformation is called **rigid.** That is, a transformation is rigid if every image is congruent to its preimage.

Excerpts from Larson, Roland (1998). *Geometry: An Integrated Approach*. Evanston, IL. McDougal LIttell.

Example 2 *Identifying Rigid Transformations*

Which of the following transformations appear to be rigid?



Solution

a. This transformation appears to be rigid. The image is congruent to the preimage.

b. This transformation appears to be not rigid. The image is not congruent to the preimage; it is smaller.

c. This transformation appears to be not rigid. The image is not congruent to the preimage; it is fatter.

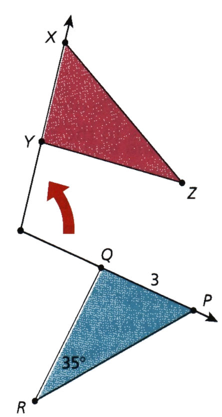
d. This transformation appears to be rigid. The image is congruent to the preimage. ■

A rigid transformation is called an **isometry.** (The Greek phrase *isos metrom* means "equal measure.")

Definition of Isometry

A transformation in the plane is an **isometry** if it preserves lengths. (That is, every segment is congruent to its image.)

It can be proved that isometries not only preserve lengths, they also preserve angle measures, parallel lines, and betweenness of points.



Example 3 *Preserving Distance and Angle Measure*

In the figure at the left, $\triangle PQR$ is mapped onto $\triangle XYZ$. The mapping is a rotation. Find the length of $\overline{XY}$ and the measure of $\angle Z$.

Solution Because a rotation is an isometry, the two triangles are congruent, so $XY = PQ = 3$ and $m\angle Z = m\angle R = 35°$. ■

In Example 3, note that the statement "$\triangle PQR$ is mapped onto $\triangle XYZ$" implies the correspondence $P \rightarrow X$, $Q \rightarrow Y$, and $R \rightarrow Z$.

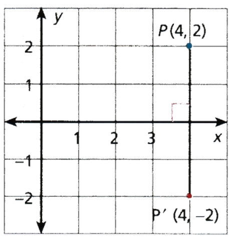
What you should learn:

Goal 1 How to use properties of reflections

Goal 2 How to relate reflections and line symmetry

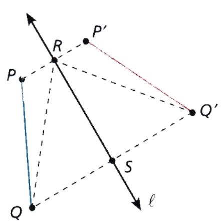
Why you should learn it:

You can use reflections to solve real-life problems, such as building a kaleidoscope.



Strategies

Consider all cases

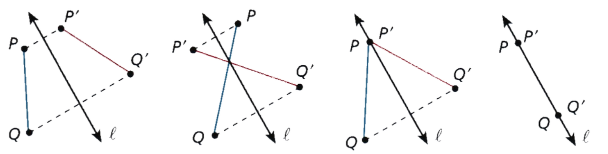


Goal 1 Using Reflections

A **reflection** in a line ℓ is a transformation that maps every point P in the plane to a point P', so that the following properties are true.

1. If P is not on ℓ, then ℓ is the perpendicular bisector of $\overline{PP'}$.
2. If P is on ℓ, then $P = P'$.

For instance, suppose the points in a coordinate plane are reflected in the x-axis. In general, every point (x, y) is mapped onto the point $(x, -y)$, so $P(4, 2)$ is mapped onto $P'(4, -2)$, as shown at the left. Notice that the x-axis is the perpendicular bisector of $\overline{PP'}$.



In the four figures above, the segment $\overline{PQ}$ is reflected in line ℓ. The image of $\overline{PQ}$ is $\overline{P'Q'}$. In each case, note that both segments appear to have the same length.

Theorem 7.1 A reflection is an isometry.

Proof: The four cases that must be proved are illustrated above. Refer to the figure at the left for Case 1.

Given: A reflection in ℓ maps P onto P' and Q onto Q'.
Prove: $PQ = P'Q'$

Plan for Proof: Suppose that P and Q are on the same side of line ℓ. Draw $\overline{PP'}$ and $\overline{QQ'}$, intersecting line ℓ at R and S. Draw $\overline{RQ}$ and $\overline{RQ'}$. Show that $\triangle RSQ \cong \triangle RSQ'$ by SAS. Then show that $\triangle RQP \cong \triangle RQ'P'$ by SAS. Finally, use CPCTC to conclude that $PQ = P'Q'$.

You are asked to prove this case and another one in Exercises 26 and 27.

Excerpts from Larson, Roland (1998). *Geometry: An Integrated Approach*. Evanston, IL. McDougal LIttell.

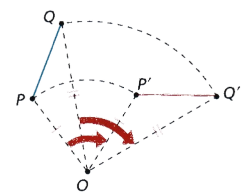
What you should learn:

Goal 1 How to use properties of rotations

Goal 2 How to relate rotations and rotational symmetry

Why you should learn it:

You can use rotations to solve real-life problems, such as determining the symmetry of a clock face.



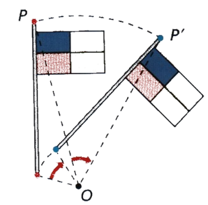
This figure illustrates Case 1. In Case 2, O, P, and Q are collinear. In Case 3, P and O are the same point.

Goal 1 Using Rotations

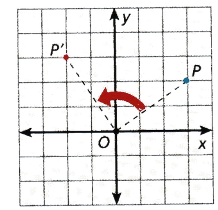
A **rotation** about a point O through x degrees ($x°$) is a transformation that maps every point P in the plane to a point P', so that the following properties are true.

1. If P is not point O, then $PO = P'O$ and $m\angle POP' = x°$.
2. If P is point O, then $P = P'$.

In the left figure below, the flag is rotated 45° clockwise about the point O. In the right figure, the point in a coordinate plane is rotated 90° counterclockwise about the origin.



Clockwise rotation of 45°



Counterclockwise rotation of 90°

Theorem 7.2 A rotation is an isometry.

Proof of Case 1: Refer to the figure at the left for Case 1.

Given: A rotation about O maps P onto P' and Q onto Q'.
Prove: $PQ = P'Q'$

Plan for Proof: By the definition of a rotation,

$$OP = OP' \text{ and } OQ = OQ'.$$

Also, by definition of a rotation, $m\angle POP' = m\angle QOQ'$. You can use the Subtraction Property of Equality to conclude that

$$m\angle POQ = m\angle P'OQ'.$$

This allows you to use the SAS Congruence Postulate to conclude that $\triangle POQ \cong \triangle P'OQ'$. Finally, using CPCTC, it follows that $PQ = P'Q'$. ■