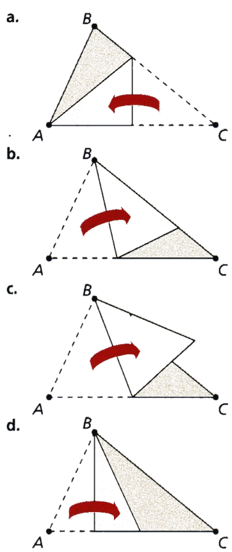
What you should learn:

Goal 1 How to identify special segments in a triangle

Goal 2 How to use properties of special segments to solve problems in geometry and in real life

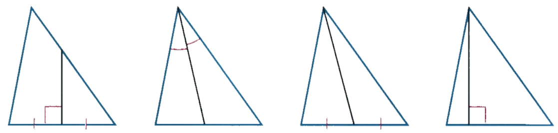
Why you should learn it:

Using the properties of special segments related to triangles can help you answer questions about triangular objects, such as how to find the balancing point of a triangular model.



Goal 1 Identifying Special Segments

In Chapter 4, you studied several properties of the sides and angles of triangles. In this lesson, you will study four other segments related to triangles: perpendicular bisectors, angle bisectors, medians, and altitudes.



Perpendicular Bisector | *Angle Bisector* | *Median* | *Altitude*

The first three segments lie in a triangle's interior. Their endpoints are points of the triangle. A **perpendicular bisector of a triangle** is a segment that is part of a perpendicular bisector of one of the sides. An **angle bisector of a triangle** is a segment that bisects one of the angles of the triangle.

A **median** is a segment whose endpoints are a vertex and the midpoint of the opposite side.

An **altitude** is a segment from a vertex that is perpendicular to the opposite side or to the line containing the opposite side. An altitude may lie inside *or* outside the triangle.

Angle bisectors, medians, and altitudes always have a vertex as one endpoint. Perpendicular bisectors may or may not have a vertex as one endpoint.

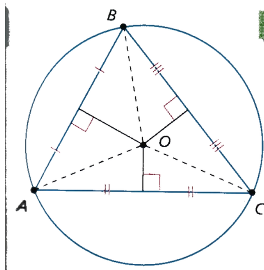
LESSON INVESTIGATION

Investigating Special Segments of a Triangle

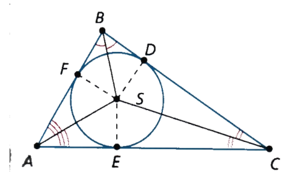
Cut four large, acute, scalene triangles out of paper. Label the vertices of each triangle as A, B, and C. Explain how to fold the triangles to produce the following special segments.

a. The perpendicular bisector of side $\overline{AC}$
b. The angle bisector of $\angle B$
c. The median from $\angle B$ to $\overline{AC}$
d. The altitude from $\angle B$ to $\overline{AC}$

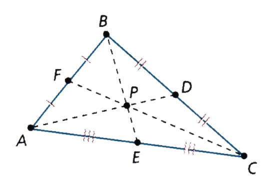
Excerpts from Larson, Roland (1998). *Geometry: An Integrated Approach*. Evanston, IL. McDougal LIttell.



The circumcenter is the center of the circumscribed circle that passes through the vertices of the triangle. That is, AO = BO = CO.



The incenter is equidistant from the three sides. It is the center of the inscribed circle of the triangle. That is, SE = SF = SD.



The centroid is two-thirds the distance from each vertex to the opposite side. For instance, $PC = \frac{2}{3}(CF)$.

Goal 2 Using Concurrency Properties

Every triangle has three perpendicular bisectors, three angle bisectors, three medians, and three altitudes. In the following investigation, you are asked to discover a special property of each of these sets of special segments.

LESSON INVESTIGATION

Investigating Special Segments of a Triangle

Use the same four paper triangles that you folded on the previous page. In each case, compare your results to those of your classmates. What can you conclude?

a. Fold the other two perpendicular bisectors.
b. Fold the other two angle bisectors.
c. Fold the other two medians.
d. Fold the other two altitudes.

In the above investigation, you may have discovered that each of the sets of special segments is **concurrent.** That is, each set intersects in a single point.

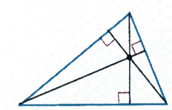
Theorem 5.5 Concurrency Properties

The lines containing the perpendicular bisectors of a triangle are concurrent. Their common point is the **circumcenter** of the triangle. The circumcenter is equidistant from the vertices of the triangle.

The angle bisectors of a triangle are concurrent. Their common point is the **incenter** of the triangle. The incenter is equidistant from the three sides of the triangle.

The medians of a triangle are concurrent. Their common point is the **centroid** of the triangle. The centroid is two-thirds of the distance from each vertex to the midpoint of the opposite side.

The lines containing the altitudes of a triangle are concurrent. Their common point is the **orthocenter** of the triangle.



Outline of Proof for Centroid: Use a coordinate proof in which the vertices of the triangle are $A(-r, 0)$, $B(s, t)$, and $C(r, 0)$. The intersection of the three medians can be shown to be $(\frac{1}{3}s, \frac{1}{3}t)$. Use the Distance Formula to show that this point is two-thirds of the distance from each vertex to the midpoint of the opposite side. ■

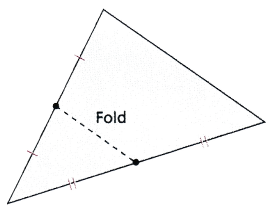
What you should learn:

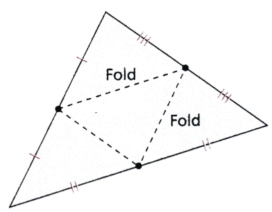
Goal 1 How to identify and construct the midsegments of a triangle

Goal 2 How to use properties of midsegments to solve real-life problems

Why you should learn it:

You can use the midsegments of a triangle to answer questions about triangles that occur in real-life situations such as building a roof framework.





Connections **Technology**

Goal 1 Constructing Midsegments

You already know how to construct, identify, and use four special segments of triangles: perpendicular bisectors, angle bisectors, medians, and altitudes.

In this lesson, you will study another special segment—a **midsegment.** A midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle.

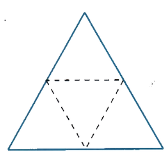
LESSON INVESTIGATION

▪ *Investigating Properties of Midsegments*

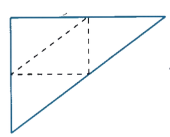
To perform this investigation, you need a triangle cut out of paper. The triangle may be of any type.

1. Find the midpoints of any two sides of the triangle.
2. Fold the segment that contains these two midpoints. (This segment is a midsegment of the triangle.)
3. Unfold the triangle and state any observations that look true about the triangle and the midsegment.
4. Fold and unfold the remaining two midsegments of the triangle.
5. What observations can you make regarding the three midsegments of your triangle? You should look for a relationship between the midsegments and the sides *and* a relationship between the lengths of the midsegments and the lengths of the sides.

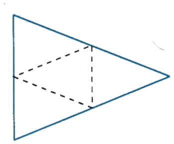
If you have access to a computer drawing program, use it to construct the midsegments of several other triangles (some examples are shown below). Do your constructions support the observations you made?



Equilateral triangle



Right triangle



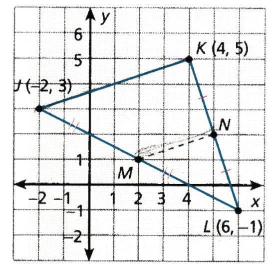
Isosceles triangle

Excerpts from Larson, Roland (1998). *Geometry: An Integrated Approach*. Evanston, IL. McDougal LIttell.

Goal 2 Using Properties of Midsegments

Example 1 *Illustrating the Midsegment Theorem*

Show that the midsegment $\overline{MN}$ is parallel to the side $\overline{JK}$ and half its length.



Solution Begin by using the Midpoint Formula to find the coordinates of M and N.

$$M = \left(\frac{-2+6}{2}, \frac{3+(-1)}{2}\right) = (2, 1)$$

$$N = \left(\frac{4+6}{2}, \frac{5+(-1)}{2}\right) = (5, 2)$$

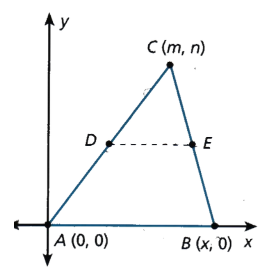
The slopes of $\overline{JK}$ and $\overline{MN}$ are as follows.

$$\text{Slope of } \overline{JK} = \frac{5-3}{4-(-2)} = \frac{2}{6} = \frac{1}{3} \qquad \text{Slope of } \overline{MN} = \frac{2-1}{5-2} = \frac{1}{3}$$

Because $\overline{JK}$ and $\overline{MN}$ have the same slope, they must be parallel. You can use the Distance Formula to show that the length of $\overline{MN}$ is $\sqrt{10}$ and the length of $\overline{JK}$ is $\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$. Therefore, MN is half of JK. ■

Theorem 5.6 Midsegment Theorem

The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half its length.



Proof: For this coordinate proof, begin by placing the coordinate system conveniently. Choose the coordinates of the vertices to be $A(0, 0)$, $B(x, 0)$, and $C(m, n)$. You can use the Midpoint Formula to find the coordinates of D and E.

Coordinates of D

$$\left(\frac{m+0}{2}, \frac{n+0}{2}\right) = \left(\frac{m}{2}, \frac{n}{2}\right)$$

Coordinates of E

$$\left(\frac{m+x}{2}, \frac{n+0}{2}\right) = \left(\frac{m+x}{2}, \frac{n}{2}\right)$$

Because D and E have the same y-coordinates, the midsegment $\overline{DE}$ has a slope of zero. Because $\overline{AB}$ also has a slope of zero, the segments $\overline{DE}$ and $\overline{AB}$ are parallel.

Because the segments $\overline{DE}$ and $\overline{AB}$ are horizontal, their lengths are given by the absolute values of the differences of their x-coordinates.

$$AB = |x - 0| = x \qquad DE = \left|\frac{m+x}{2} - \frac{m}{2}\right| = \frac{x}{2} = \frac{1}{2}x$$

The length of $\overline{DE}$ is half the length of $\overline{AB}$. A similar argument can be given for the other two midsegments. ■

Strategies **Choosing the Axes**