

Tangents

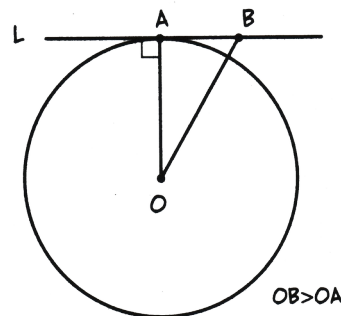
ARE THERE LINES THAT TOUCH THE CIRCLE AT ONLY ONE POINT? LINES THAT "KISS" IT, AS MATHEMATICIANS SAY? (ACTUALLY, THEY SAY "OSCULATE," BECAUSE IT HAS MORE SYLLABLES.)



Theorem 17-3. THROUGH EACH POINT ON A CIRCLE, THERE IS EXACTLY ONE LINE THAT INTERSECTS THE CIRCLE AT ONLY THAT POINT.

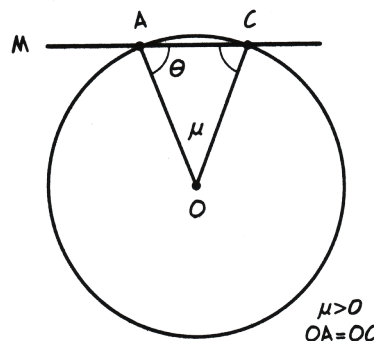
Proof. LET A BE A POINT ON THE CIRCLE WITH CENTER O . LET L BE THE LINE THROUGH A WITH $L \perp OA$. FIRST WE SHOW THAT L INTERSECTS THE CIRCLE NOWHERE ELSE; THEN WE SHOW THAT ANY OTHER LINE THROUGH A INTERSECTS THE CIRCLE AT ANOTHER POINT.

1. LET B BE ANY OTHER POINT ON L . (RULER POST.)
2. $L \perp OA$ (ASSUMED)
3. $\triangle OAB$ IS A RIGHT TRIANGLE WITH OB AS HYPOTENUSE. (DEFINITIONS)
4. $OB > OA$ (THM. 7-3)
5. B IS NOT ON THE CIRCLE. (DEF. OF CIRCLE)



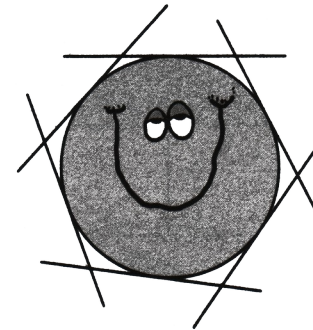
NOW LET M BE A LINE THROUGH A NOT PERPENDICULAR TO OA . M AND OA FORM AN ANGLE $\theta < 90^\circ$ ON ONE SIDE.

6. DRAW RAY OC AT AN ANGLE $\mu = 180^\circ - 2\theta$ WITH OA ON THE SAME SIDE OF THE RADIUS AS $\angle \theta$, INTERSECTING M AT C .
7. $\angle ACO = 180^\circ - (\theta + \mu) = \theta$ (THM. 10-4)
8. $\triangle AOC$ IS ISOSCELES, WITH $OA = OC$. (THM. 6-2)
9. C IS ON THE CIRCLE, AND M INTERSECTS THE CIRCLE TWICE. (DEF. OF CIRCLE)



Definition. A LINE THAT TOUCHES A CIRCLE AT A SINGLE POINT IS CALLED A TANGENT TO THE CIRCLE.

THEOREM 17-3 SAYS THAT THERE IS EXACTLY ONE "KISSING" TANGENT AT EVERY POINT OF THIS LUCKY CIRCLE: THE PERPENDICULAR TO THE RADIUS AT THAT POINT.

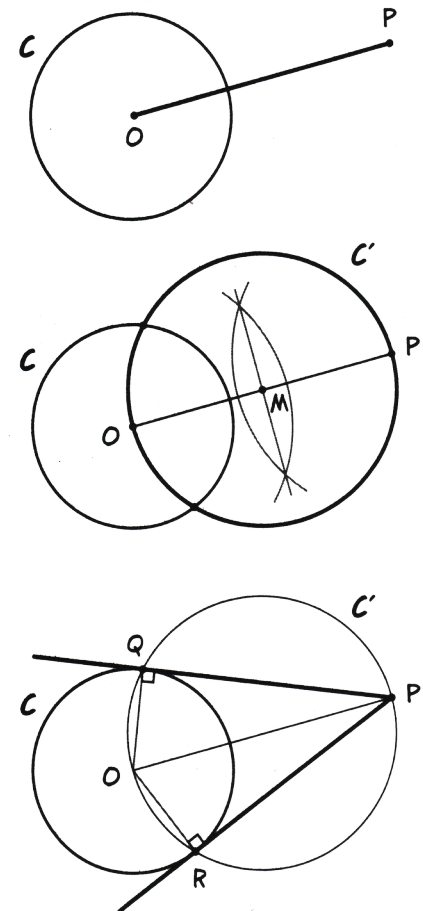


THE NEXT THEOREM SAYS HOW TO "AIM" A TANGENT FROM ANY POINT IN THE PLANE OUTSIDE THE CIRCLE.

Theorem 17-4. GIVEN A POINT P OUTSIDE A CIRCLE C , THERE ARE TWO TANGENTS CONTAINING P .

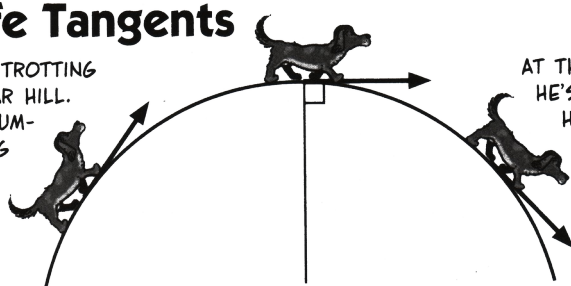
Proof.

1. FIND O , THE CENTER OF C , AND DRAW OP . (CONSTR., P. 87; POST. 2)
2. FIND M , THE MIDPOINT OF OP . (CONSTR., P. 85)
3. DRAW A CIRCLE C' CENTERED AT M WITH RADIUS OM . (DEF. OF CIRCLE)
4. OMP IS A DIAMETER OF C' . (DEF. OF DIAMETER)
5. LET C AND C' INTERSECT AT POINTS Q AND R . (POST. 6)
6. $\angle OQP$ AND $\angle ORP$ ARE RIGHT ANGLES. (COR. 17-2.4)
7. PR AND QR ARE TANGENT TO C . (THM. 17-3)



Real-Life Tangents

HERE'S JASPER TROTTING OVER A CIRCULAR HILL. NEARING THE SUMMIT, HE'S GOING UPHILL, AND AFTER PASSING IT, HE'S GOING DOWN.



AT THE VERY TOP, HE'S MOVING EXACTLY HORIZONTALLY, PERPENDICULAR TO THE RADIUS THERE.

OR IMAGINE A ROTATING CIRCLE, LIKE A FERRIS WHEEL. THE SAME REASONING SUGGESTS THAT

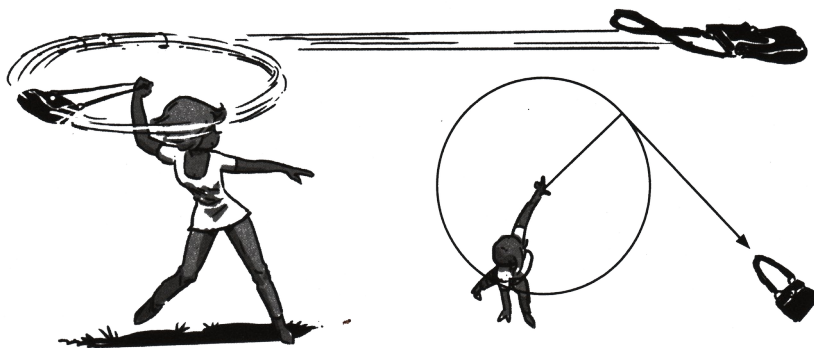


THE TANGENT IS THE DIRECTION OF THE CIRCLE AT THAT POINT.

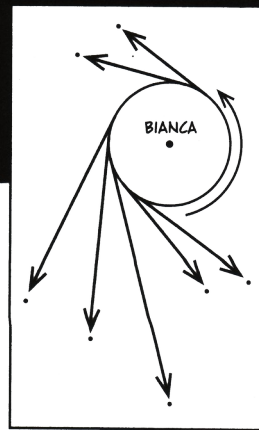
WELL, THAT'S A LITTLE HARD TO GRASP...



IN PRACTICAL TERMS, IT MEANS THIS: IF BIANCA SWINGS HER PURSE AROUND IN A CIRCLE AND THEN LETS IT GO, IT WILL LITERALLY "FLY OFF ON A TANGENT," FOLLOWING THE CIRCLE'S TANGENT LINE AT THE POINT OF RELEASE.

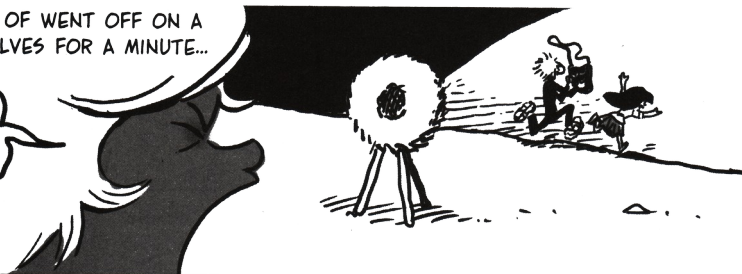


BY THEOREM 17-3, A CIRCLE'S TANGENTS PASS THROUGH EVERY POINT IN THE PLANE OUTSIDE THE CIRCLE, SO WITH GOOD AIM AND A STRONG ARM, BIANCA SHOULD BE ABLE TO HIT ANY TARGET ANYWHERE!



OKAY... WE KIND OF WENT OFF ON A TANGENT OURSELVES FOR A MINUTE...

HEY! GIVE THAT BACK!



MOVING RIGHT ALONG, THEN...

