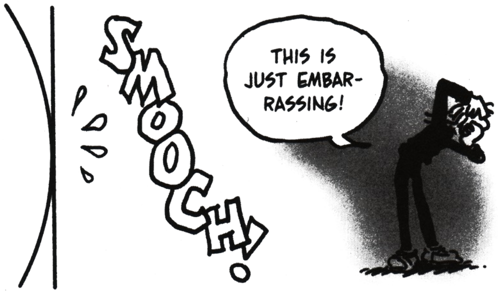
Tangents

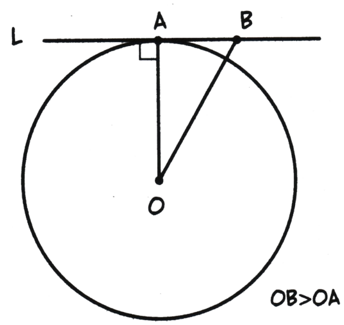
ARE THERE LINES THAT TOUCH THE CIRCLE AT ONLY ONE POINT? LINES THAT "KISS" IT, AS MATHEMATICIANS SAY? (ACTUALLY, THEY SAY "OSCULATE," BECAUSE IT HAS MORE SYLLABLES.)



Theorem 17-3. THROUGH EACH POINT ON A CIRCLE, THERE IS **EXACTLY ONE LINE** THAT INTERSECTS THE CIRCLE AT ONLY THAT POINT.

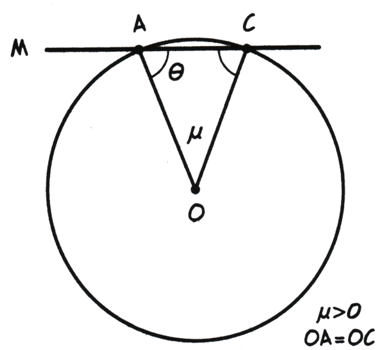
Proof. LET **A** BE A POINT ON THE CIRCLE WITH CENTER **O**. LET **L** BE THE LINE THROUGH **A** WITH **L⊥OA**. FIRST WE SHOW THAT L INTERSECTS THE CIRCLE NOWHERE ELSE; THEN WE SHOW THAT ANY OTHER LINE THROUGH **A** INTERSECTS THE CIRCLE AT ANOTHER POINT.

1. LET **B** BE ANY OTHER POINT ON L.	(RULER POST.)
2. L⊥OA	(ASSUMED)
3. △OAB IS A RIGHT TRIANGLE WITH OB AS HYPOTENUSE.	(DEFINITIONS)
4. OB>OA	(THM. 7-3)
5. B IS NOT ON THE CIRCLE.	(DEF. OF CIRCLE)



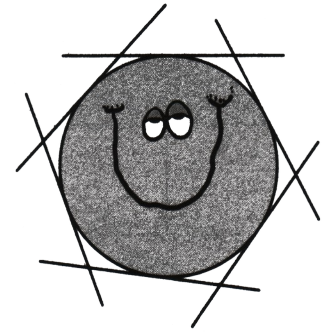
NOW LET M BE A LINE THROUGH A **NOT** PERPENDICULAR TO OA. M AND OA FORM AN ANGLE $\theta < 90°$ ON ONE SIDE.

6. DRAW RAY $\overline{OC}$ AT AN ANGLE $\mu = 180° - 2\theta$ WITH OA ON THE SAME SIDE OF THE RADIUS AS $\angle\theta$, INTERSECTING M AT C.	
7. $\angle ACO = 180° - (\theta + \mu)$ $= \theta$	(THM. 10-4)
8. △AOC IS ISOSCELES, WITH OA = OC.	(THM. 6-2)
9. C IS ON THE CIRCLE, AND M INTERSECTS THE CIRCLE TWICE. ▮	(DEF. OF CIRCLE)



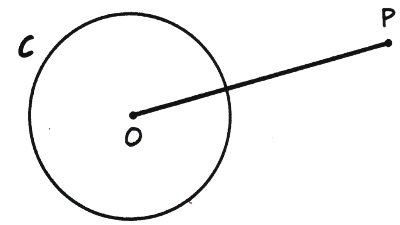
Definition. A LINE THAT TOUCHES A CIRCLE AT A SINGLE POINT IS CALLED A **TANGENT** TO THE CIRCLE.

THEOREM 17-3 SAYS THAT THERE IS EXACTLY ONE "KISSING" TANGENT AT EVERY POINT OF THIS LUCKY CIRCLE: THE PERPENDICULAR TO THE RADIUS AT THAT POINT.



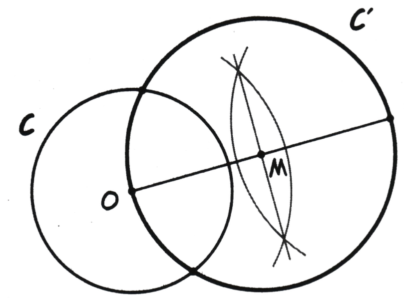
THE NEXT THEOREM SAYS HOW TO "AIM" A TANGENT FROM ANY POINT IN THE PLANE OUTSIDE THE CIRCLE.

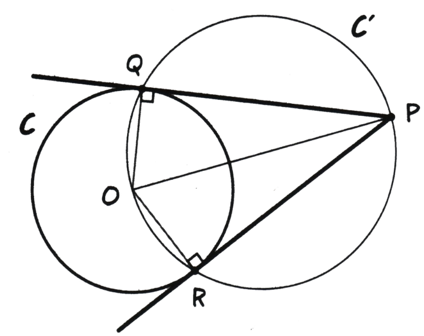
Theorem 17-4. GIVEN A POINT P OUTSIDE A CIRCLE **C**, THERE ARE TWO TANGENTS CONTAINING P.

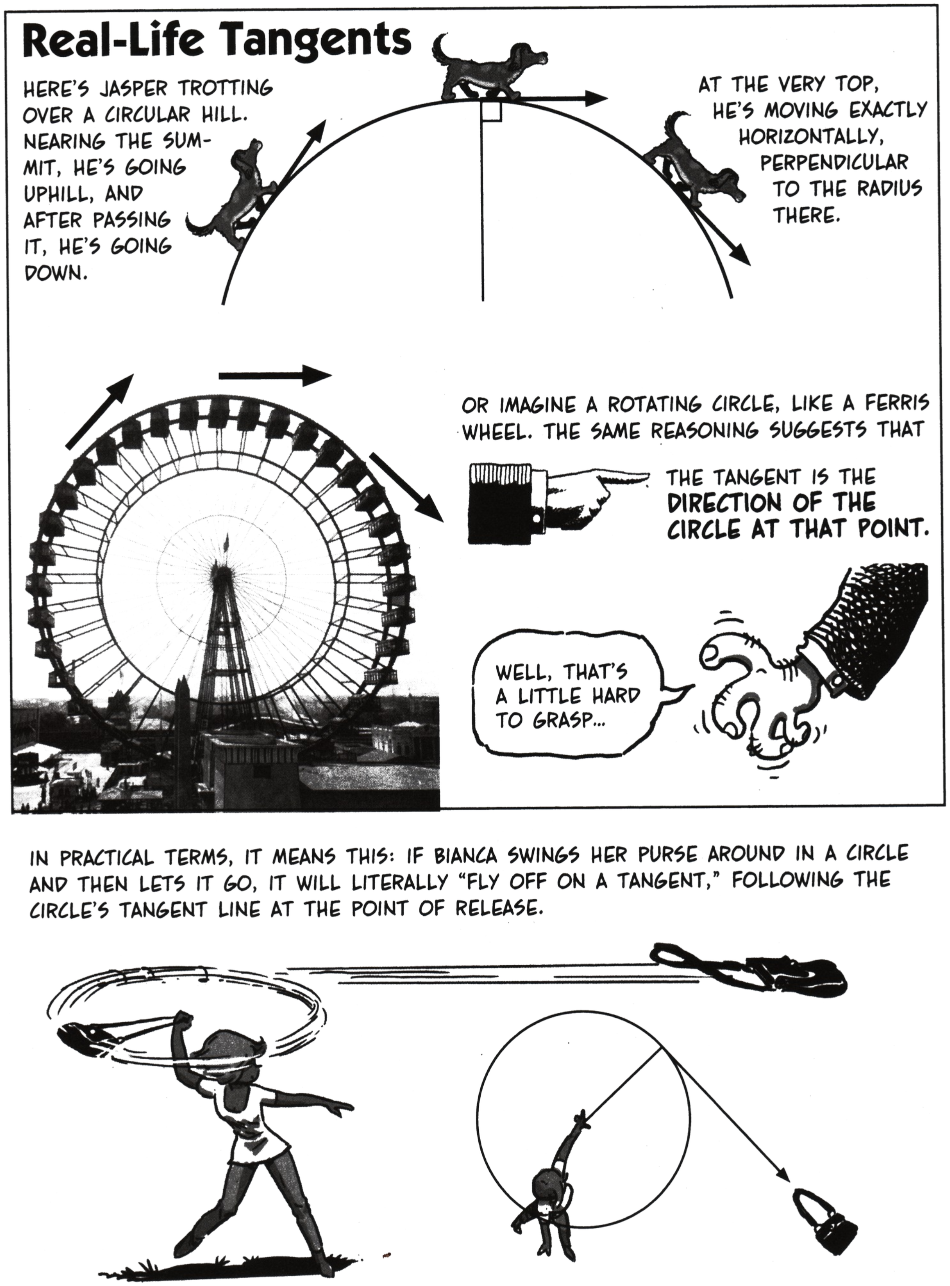


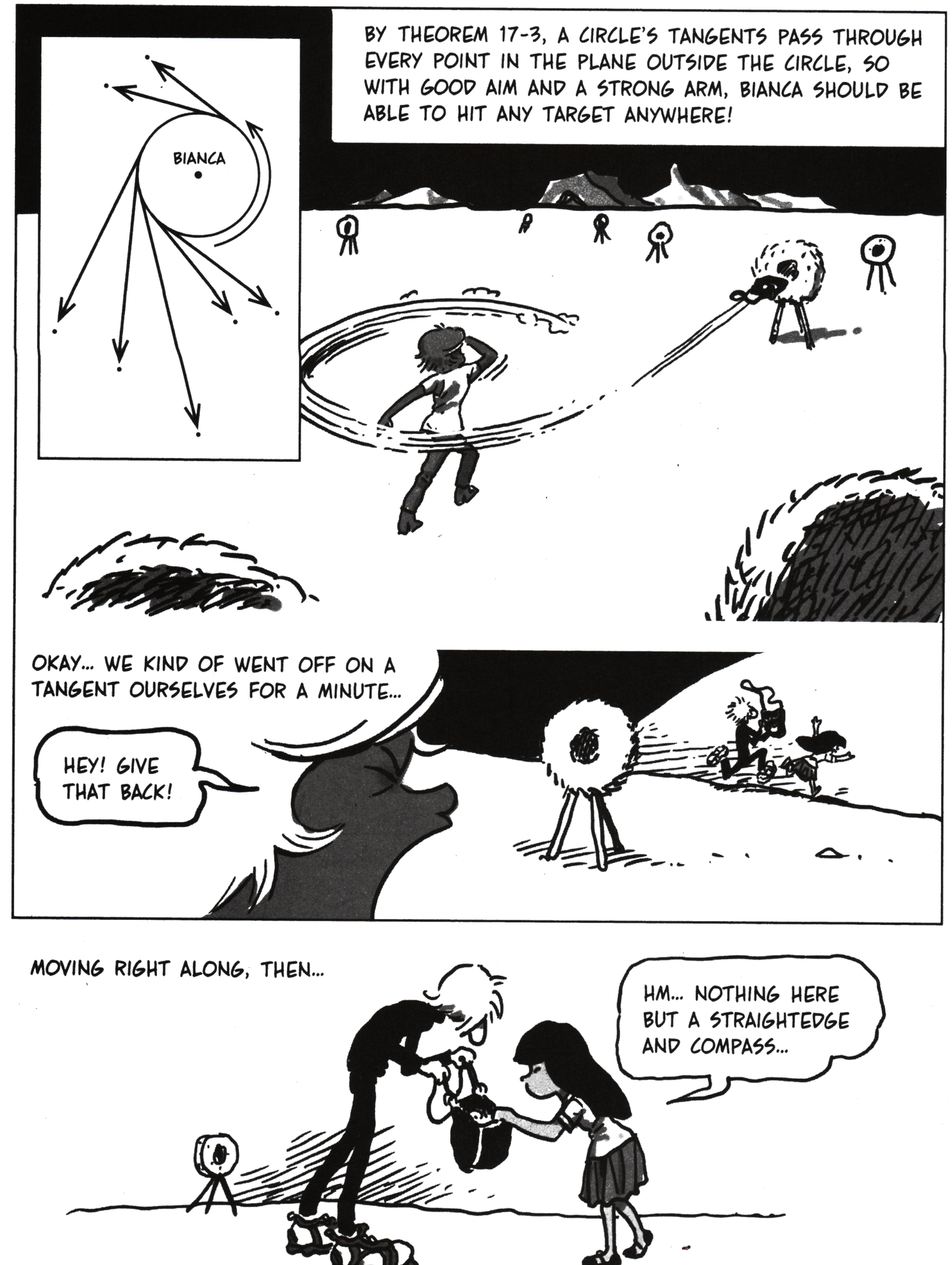
Proof.

1. FIND O, THE CENTER OF **C**, AND DRAW OP.	(CONSTR., P. 87; POST. 2)
2. FIND M, THE MIDPOINT OF OP.	(CONSTR., P. 85)
3. DRAW A CIRCLE **C′** CENTERED AT M WITH RADIUS OM.	(DEF. OF CIRCLE)
4. OMP IS A DIAMETER OF **C′**.	(DEF. OF DIAMETER)
5. LET **C** AND **C′** INTERSECT AT POINTS Q AND R.	(POST. 6)
6. ∠OQP AND ∠ORP ARE RIGHT ANGLES.	(COR. 17-2.4)
7. PR AND QR ARE TANGENT TO **C**. ▮	(THM. 17-3)









Excerpts from Gonick, Larry (2024). *The Cartoon Guide to Geometry*. New York, NY. Harper Collins.