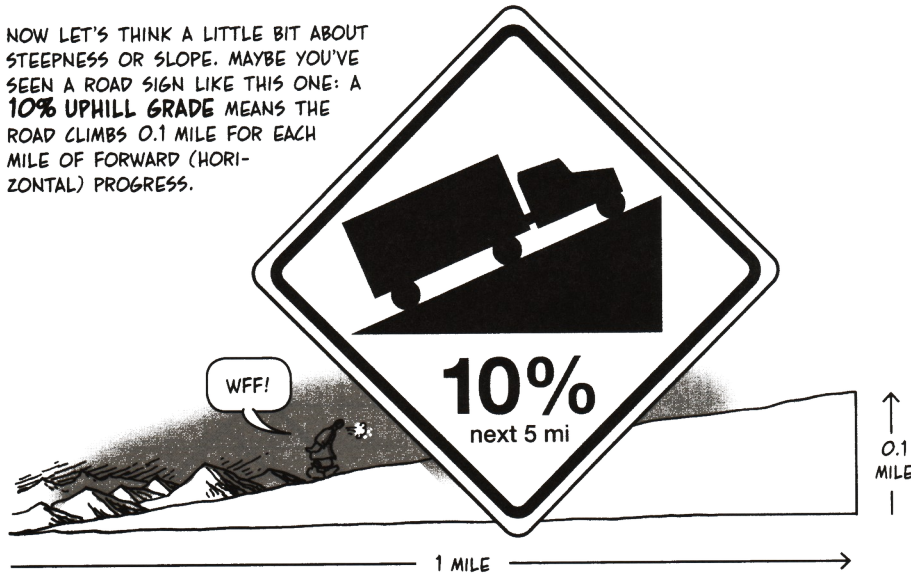


NOW LET'S THINK A LITTLE BIT ABOUT STEEPNESS OR SLOPE. MAYBE YOU'VE SEEN A ROAD SIGN LIKE THIS ONE: A **10% UPHILL GRADE** MEANS THE ROAD CLIMBS 0.1 MILE FOR EACH MILE OF FORWARD (HORIZONTAL) PROGRESS.



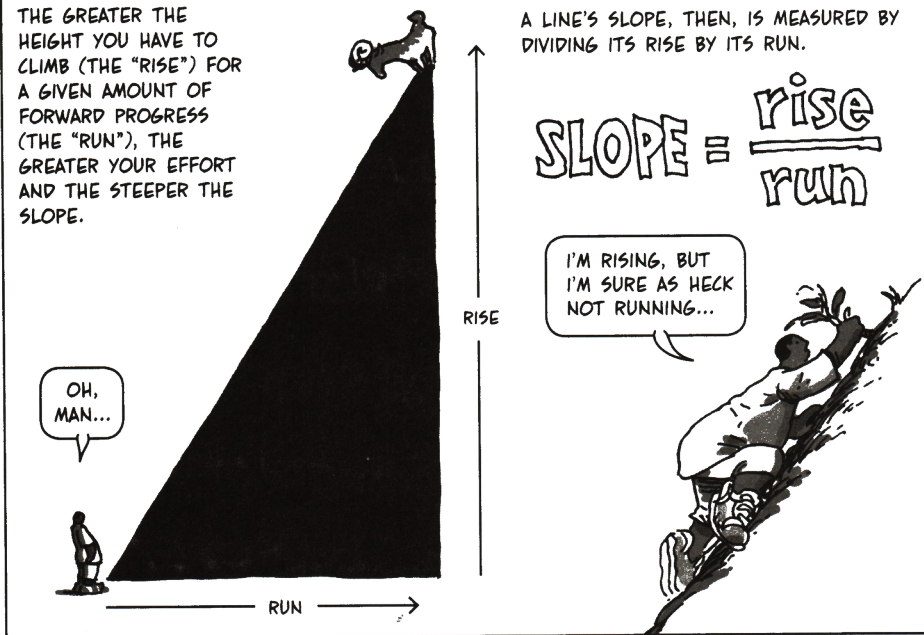
THE GREATER THE HEIGHT YOU HAVE TO CLIMB (THE "RISE") FOR A GIVEN AMOUNT OF FORWARD PROGRESS (THE "RUN"), THE GREATER YOUR EFFORT AND THE STEEPER THE SLOPE.

A LINE'S SLOPE, THEN, IS MEASURED BY DIVIDING ITS RISE BY ITS RUN.

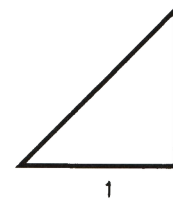
$$\text{SLOPE} = \frac{\text{rise}}{\text{run}}$$

I'M RISING, BUT I'M SURE AS HECK NOT RUNNING...

OH, MAN...



IN THE USA, INTERSTATE HIGHWAY GRADES AREN'T SUPPOSED TO EXCEED 6%, BUT LOCAL ROADS MAY BE STEEPER... AND OUR THOUGHT-LINES CAN BE STEEPER STILL. A GRADE OF **100%** WOULD RISE ONE MILE EVERY MILE (OR A FOOT EVERY FOOT, OR TEN METERS EVERY TEN METERS, ETC).



GRADE = 100%

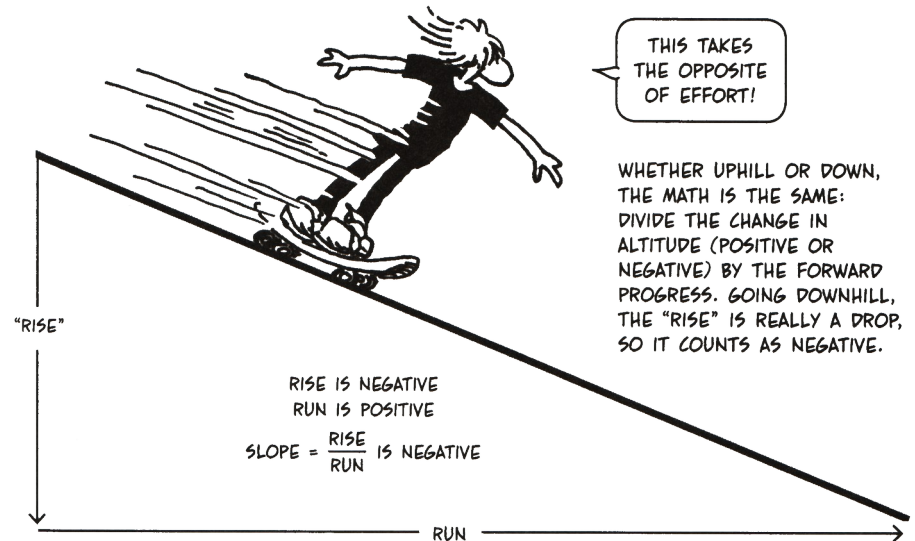
$$\text{SLOPE} = \frac{1}{1} = 1$$

AND WE CAN GO EVEN STEEPER... THERE'S NO LIMIT ON THE SLOPE!

WELL, I HAVE MY LIMITS...



SLOPE CAN GO DOWNHILL, TOO, AND THE IDEA IS THE SAME—THE FARTHER YOU DROP FOR EACH UNIT OF FORWARD PROGRESS, THE STEEPER IT IS, IN A **NEGATIVE SENSE**.



THIS TAKES THE OPPOSITE OF EFFORT!

WHETHER UPHILL OR DOWN, THE MATH IS THE SAME: DIVIDE THE CHANGE IN ALTITUDE (POSITIVE OR NEGATIVE) BY THE FORWARD PROGRESS. GOING DOWNHILL, THE "RISE" IS REALLY A DROP, SO IT COUNTS AS NEGATIVE.

NOW LET'S SEE HOW THIS LOOKS USING ALGEBRA.

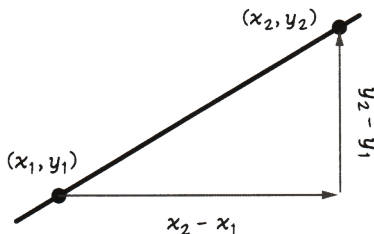
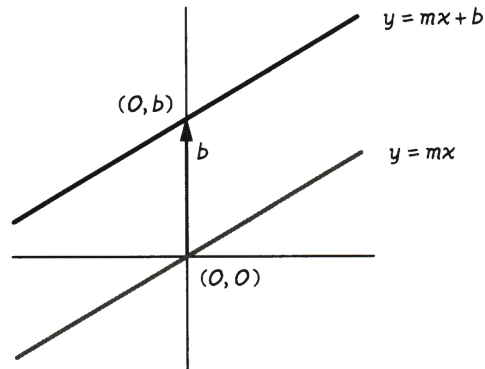
Slope and Intercept

GIVEN AN EQUATION $y = mx + b$, WHAT DO THE NUMBERS m AND b TELL US ABOUT THE GRAPH? START WITH b .

WHEN $x = 0$, THE EQUATION SAYS

$$y = m(0) + b \\ = b$$

SO THE POINT $(0, b)$ IS ON THE LINE. IN OTHER WORDS, b IS WHERE THE LINE CROSSES THE y -AXIS. THE NUMBER b IS CALLED THE y -INTERCEPT OF THE LINE.



AND NOW m : WE SAW ON P. 99 THAT m HAS SOMETHING TO DO WITH A LINE'S SLOPE, SO LET'S CALCULATE THE SLOPE. WE TAKE TWO POINTS ON THE LINE—ANY TWO POINTS—AND DIVIDE THE RISE BY THE RUN. IF THE POINTS HAVE COORDINATES (x_1, y_1) AND (x_2, y_2) ,* THEN THE RISE IS $y_2 - y_1$ AND THE RUN IS $x_2 - x_1$.

LYING ON THE LINE, BOTH POINTS' COORDINATES SATISFY THE EQUATION.

$$y_1 = mx_1 + b \\ y_2 = mx_2 + b$$

SUBTRACT y_1 FROM y_2 TO FIND THE RISE.

$$y_2 - y_1 = mx_2 - mx_1 \quad (b \text{ CANCELS}) \\ = m(x_2 - x_1) \quad (\text{DISTRIBUTIVE LAW})$$

IN WORDS: RISE OVER RUN EQUALS m !



SINCE $x_2 - x_1$ ISN'T ZERO, WE CAN DIVIDE BOTH SIDES BY THIS NUMBER TO GET THE RISE OVER THE RUN:

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

THAT MONSTER ON THE LEFT IS CALLED THE **DIFFERENCE QUOTIENT**. THIS EQUATION SAYS THAT THE DIFFERENCE QUOTIENT IS **ALWAYS m** , FOR ANY TWO POINTS ON THE LINE. **m IS THE SLOPE!!**

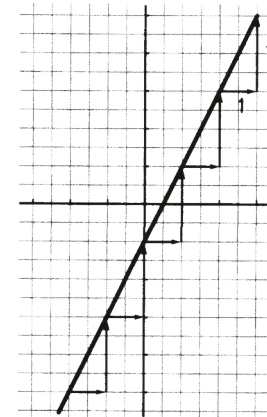
*READ AS "EX-ONE," "WYE-ONE," "EX-TWO," "WYE-TWO." NO ARITHMETIC IS IMPLIED BY THE LITTLE SUBSCRIPT NUMBERS $_1$ AND $_2$. THEY ARE SIMPLY LABELS TO IDENTIFY WHICH OF THE TWO DIFFERENT POINTS "OWN" THE COORDINATES. x_1 IS THE x -COORDINATE OF THE FIRST POINT, AND SO ON.

Example. HERE IS THE GRAPH OF $y = 2x - 1$ WITH SOME OF ITS POINTS' COORDINATES LISTED IN A TABLE. NOTE THAT EVERY TIME x INCREASES BY 1, y INCREASES BY 2, THE COEFFICIENT OF x .

IN FACT, ANY TWO POINTS ON THIS LINE HAVE A DIFFERENCE QUOTIENT OF 2. LET'S TRY $(-2, -5)$ AND $(2, 3)$.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-5)}{2 - (-2)} \\ = \frac{8}{4} = 2$$

TRY THIS WITH OTHER PAIRS OF POINTS. IF YOU GET ANYTHING OTHER THAN 2, YOU MADE A MISTAKE!



x	$2x - 1$
-3	-7
-2	-5
-1	-3
0	-1
1	1
2	3
3	5



THE EQUATION $y = mx + b$ IS IN **SLOPE-INTERCEPT form.**

WE CAN READ A GRAPH'S SLOPE AND y -INTERCEPT DIRECTLY FROM THIS EQUATION. IT TELLS US HOW STEEP THE GRAPH IS, AND HOW FAR ABOVE OR BELOW THE ORIGIN IT PASSES. IT ALSO DIRECTLY GIVES US THE VALUE OF y FOR EACH VALUE OF x .

SUCH A BEAUTIFUL, BEAUTIFUL FORM...

WHY, THANKS!



Example. GRAPH THE EQUATION

$$6x - 2y = 5$$

AND FIND ITS SLOPE AND y -INTERCEPT.

SOLUTION: FIRST USE ALGEBRA TO REWRITE THE EQUATION IN SLOPE-INTERCEPT FORM.

$$6x - 2y = 5 \\ -2y = -6x + 5 \\ y = 3x - \frac{5}{2}$$

THE SLOPE IS 3; THE y -INTERCEPT IS $-5/2$; YOU MAKE A TABLE; I'LL DRAW THE GRAPH!

