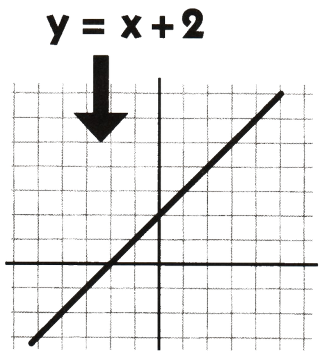
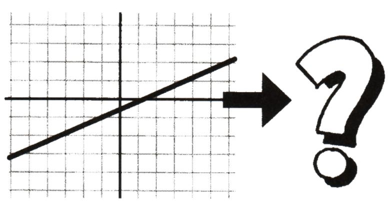
From Line to Equation

UP TO NOW, WE'VE STARTED WITH AN EQUATION AND DRAWN ITS GRAPH.



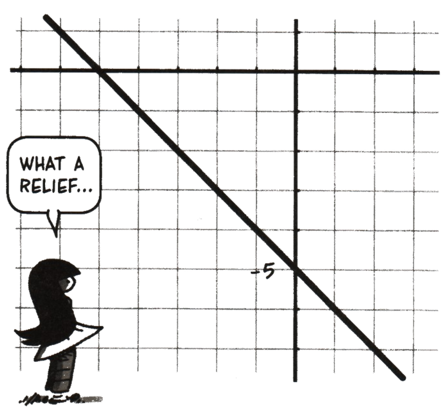
NOW LET'S GO THE OTHER WAY, FROM LINE TO EQUATION. GIVEN A LINE, CAN WE WRITE AN EQUATION WHOSE GRAPH IT IS?



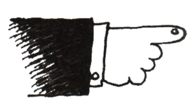
HOW MUCH DO WE NEED TO KNOW ABOUT A LINE TO WRITE ITS EQUATION?

KNOWING THE SLOPE ALONE, FOR EXAMPLE, IS NOT ENOUGH. THERE ARE HEAPS OF LINES WITH THE SAME SLOPE. HOW WOULD WE KNOW IF AN EQUATION HAD "OUR" LINE AS ITS GRAPH?

ON THE OTHER HAND, IF WE KNOW THE LINE'S SLOPE **AND** y-INTERCEPT, WE CAN WRITE ITS EQUATION IMMEDIATELY—IN SLOPE-INTERCEPT FORM, OF COURSE! FOR EXAMPLE, IF WE KNOW A LINE HAS SLOPE -1 AND y-INTERCEPT -5, ITS EQUATION SIMPLY **MUST** BE $y = -x - 5$.

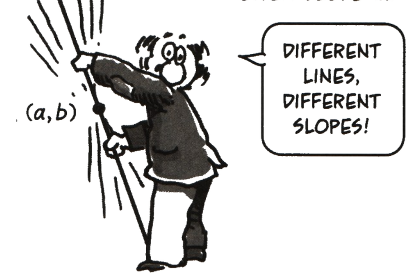


WE CAN DESCRIBE LINES IN SEVERAL DIFFERENT WAYS THAT LEAD TO EQUATIONS.



Point and Slope

THERE'S REALLY NOTHING SPECIAL ABOUT THE y-INTERCEPT. GIVEN **ANY** POINT (a, b), THERE CAN BE ONLY ONE LINE PASSING THROUGH (a, b) WITH A GIVEN SLOPE m.



THIS LINE'S EQUATION IS

$$y - b = m(x - a)$$

THIS IS CALLED THE EQUATION'S **POINT-SLOPE FORM.** TO SEE THAT THE GRAPH REALLY DOES PASS THROUGH (a, b), WE SOLVE FOR y WHEN $x = a$. PLUGGING IN a FOR x GIVES

$$y - b = m(a - a) = m \cdot 0 \quad \text{SO}$$
$$y - b = 0$$
$$y = b$$

THAT IS, IF $x = a$, THEN $y = b$, SO THE POINT (a, b) IS ON THE EQUATION'S GRAPH.

THE GRAPH HAS SLOPE m, AS YOU CAN SEE BY EXPANDING THE EXPRESSION AND COLLECTING TERMS:

$$y - b = m(x - a)$$
$$y - b = mx - ma$$
$$y = mx + (b - ma)$$

$b - ma$ IS A CONSTANT, SO THIS EQUATION IS IN SLOPE-INTERCEPT FORM, WITH SLOPE m AND y-INTERCEPT $b - ma$.

Example. FIND THE EQUATION OF A LINE PASSING THROUGH THE POINT $(7, 11)$ WITH SLOPE 6.

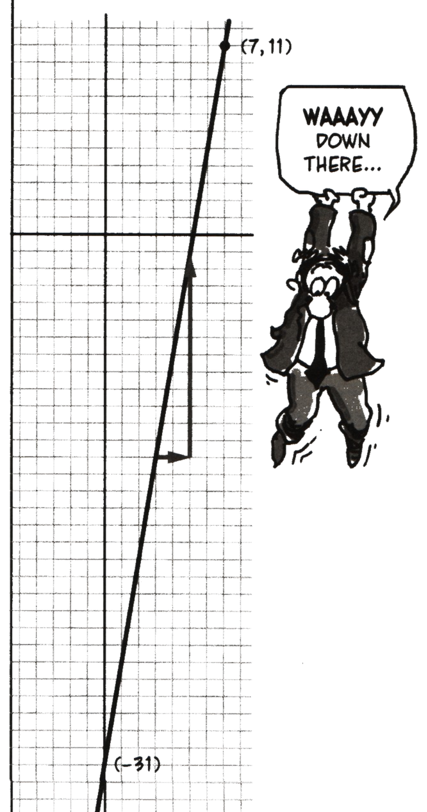
ANSWER: WE APPLY THE POINT-SLOPE FORMULA DIRECTLY AND GET

$$y - 11 = 6(x - 7)$$

WHICH CAN BE EXPANDED TO FIND THE SLOPE-INTERCEPT FORM:

$$y - 11 = 6x - 42$$
$$y = 6x - 31$$

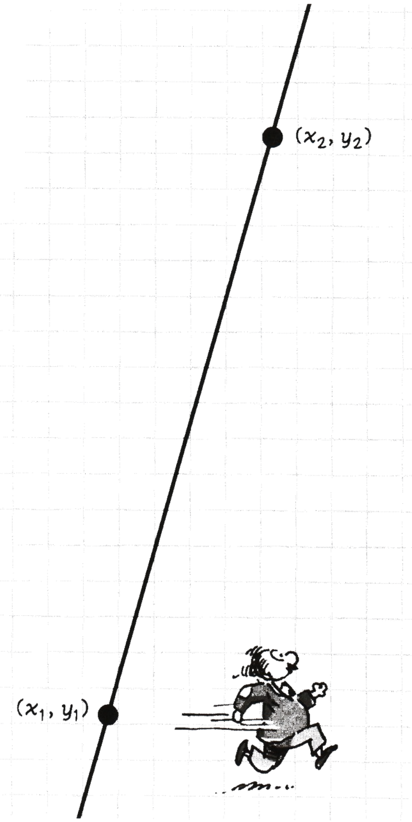
THE y-INTERCEPT IS -31.



Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.

Two Points

GIVEN TWO POINTS, YOU CAN DRAW ONE AND ONLY ONE STRAIGHT LINE THROUGH THEM. ON THE COORDINATE PLANE, IF A LINE PASSES THROUGH TWO POINTS WITH KNOWN COORDINATES (x_1, y_1) AND (x_2, y_2), WHAT IS ITS EQUATION?



FIRST FIND THE SLOPE m FROM THE DIFFERENCE QUOTIENT.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

THEN APPLY THE POINT-SLOPE FORMULA USING THIS SLOPE AND EITHER POINT. USING THE FIRST POINT (x_1, y_1), THE EQUATION IS

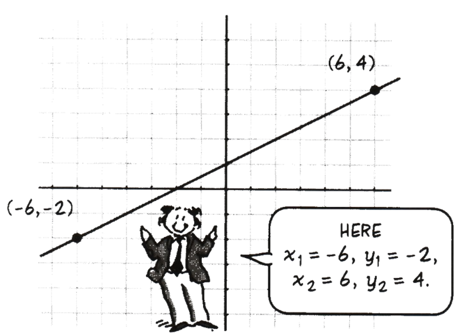
$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

USING THE SECOND POINT FORMULA GIVES AN EQUATION THAT LOOKS SLIGHTLY DIFFERENT, BUT WORKS OUT TO BE THE SAME.

$$y - y_2 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_2)$$

Example.

FIND THE EQUATION OF THE LINE PASSING THROUGH $(-6, -2)$ AND $(6, 4)$.



ANSWER: FIRST, FORM THE DIFFERENCE QUOTIENT TO FIND THE SLOPE:

$$\frac{4-(-2)}{6-(-6)} = \frac{6}{12} = \frac{1}{2}$$

PLUG THIS SLOPE INTO THE POINT-SLOPE FORMULA USING EITHER POINT. WE USE (6, 4).

$$y - 4 = \tfrac{1}{2}(x - 6)$$

$$y - 4 = \tfrac{1}{2}x - 3$$

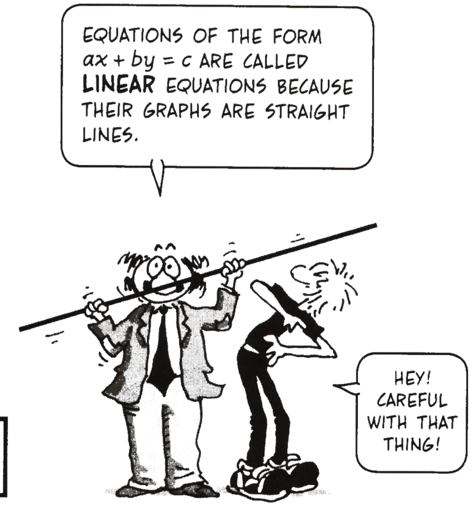
$$\mathbf{y = \tfrac{1}{2}x + 1}$$

Two Equations, Two Lines

LAST CHAPTER, WE LOOKED AT PAIRS OF EQUATIONS IN TWO VARIABLES, LIKE THESE TWO:

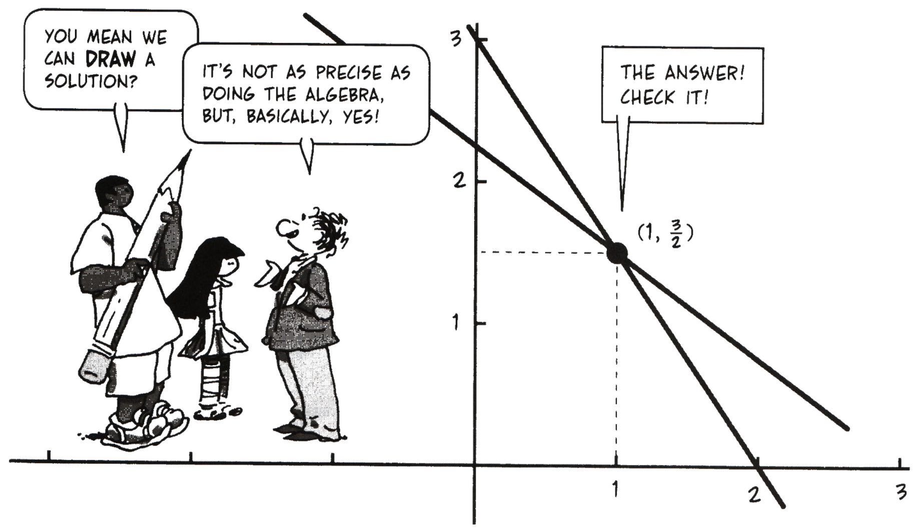
$$3x + 4y = 9$$
$$3x + 2y = 6$$

EQUATIONS LIKE THIS, IN THE FORM $ax + by = c$, ARE SAID TO BE IN **STANDARD FORM.** IF $b \neq 0$, THEY CAN EASILY BE REWRITTEN IN SLOPE-INTERCEPT FORM AND GRAPHED:



$3x + 4y = 9$	$3x + 2y = 6$
$4y = -3x + 9$	$2y = -3x + 6$
$\boxed{y = -\frac{3}{4}x + \frac{9}{4}}$	$\boxed{y = -\frac{3}{2}x + 3}$

A SOLUTION TO A PAIR OF EQUATIONS IS A PAIR OF NUMBERS (x, y) THAT SATISFIES BOTH EQUATIONS SIMULTANEOUSLY—WHICH MEANS THAT THE POINT (x, y) LIES ON **BOTH THEIR GRAPHS.** IN OTHER WORDS, THE SOLUTION (OR SOLUTIONS) TO A PAIR OF EQUATIONS IS THE POINT (OR POINTS) WHERE THEIR **GRAPHS INTERSECT!!**



Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.