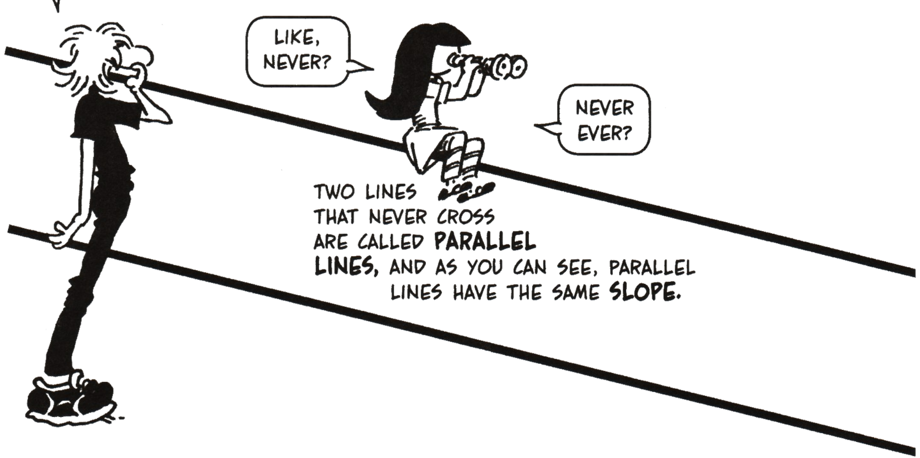
Parallel Lines

AS WE SAW IN THE LAST CHAPTER, A PAIR OF LINEAR EQUATIONS MAY HAVE **NO** SOLUTION... AND NOW WE CAN SEE WHY. THIS CAN HAPPEN ONLY WHEN THE TWO EQUATIONS' GRAPHS **NEVER CROSS.**

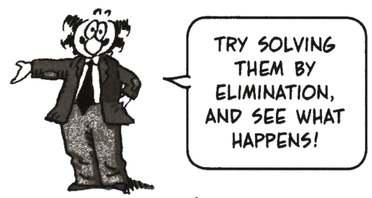


TWO LINES THAT NEVER CROSS ARE CALLED **PARALLEL LINES,** AND AS YOU CAN SEE, PARALLEL LINES HAVE THE SAME **SLOPE.**

WE CAN EASILY SEE WHETHER A PAIR OF LINEAR EQUATIONS HAVE PARALLEL GRAPHS BY PUTTING THE EQUATIONS INTO SLOPE-INTERCEPT FORM. FOR EXAMPLE:

(1) $3x + 5y = 5$

(2) $6x + 10y = 20$

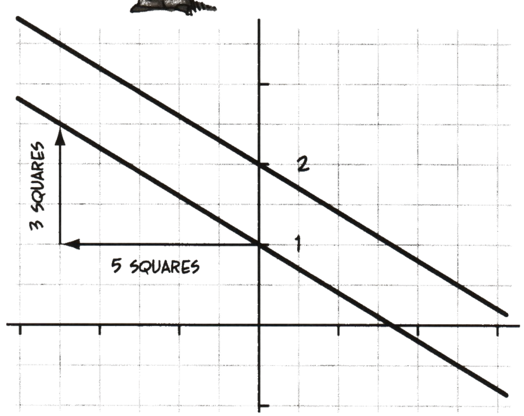


IN POINT-SLOPE FORM, THESE BECOME

(1a) $y = -\frac{3}{5}x + 1$

(2a) $y = -\frac{3}{5}x + 2$

THE y-INTERCEPTS ARE DIFFERENT, SO THE GRAPHS ARE TWO SEPARATE LINES, BUT THE SLOPES ARE THE SAME, $-3/5$, SO THE LINES ARE PARALLEL. THE EQUATIONS HAVE NO COMMON SOLUTION.

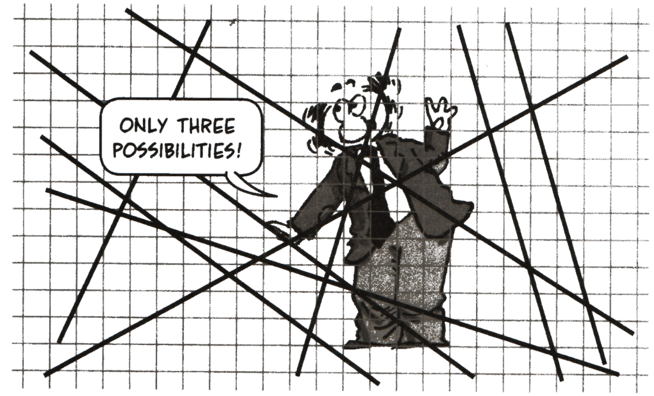


GIVEN TWO LINEAR EQUATIONS, EITHER:

1. THEY HAVE THE **SAME** GRAPH;

2. THEIR GRAPHS ARE **PARALLEL;**

3. THEIR GRAPHS **CROSS** AT A SINGLE POINT.



GIVEN TWO EQUATIONS, HOW CAN WE KNOW WHICH OPTION HOLDS TRUE? LET'S SUPPOSE a, b, c, d, e, AND f ARE SOME FIXED NUMBERS, AND THAT NEITHER b NOR d IS ZERO. (b AND d WILL BE THE COEFFICIENTS OF y IN THE TWO EQUATIONS, AND WE'LL WANT TO DIVIDE BY THEM.) WE WANT TO KNOW ABOUT THE STANDARD-FORM EQUATIONS 3 AND 4.

(3) $ax + by = e$

(4) $cx + dy = f$

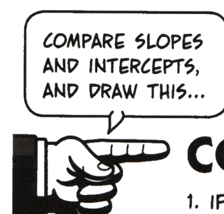
WE NEXT PUT THEM IN POINT-SLOPE FORM:

(3a) $y = -\frac{a}{b}x + \frac{e}{b}$

(4a) $y = -\frac{c}{d}x + \frac{f}{d}$

y-INTERCEPTS

SLOPES

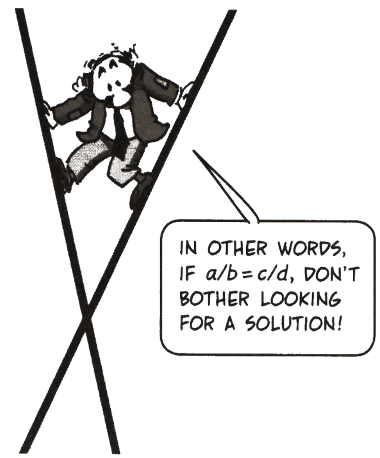


CONCLUSION:

1. IF $a/b = c/d$ **AND** $e/b = f/d$, THEN THE EQUATIONS' GRAPHS HAVE THE SAME SLOPE AND y-INTERCEPT. THEY ARE THE SAME LINE! EVERY POINT ON THIS LINE SOLVES BOTH EQUATIONS.

2. IF $a/b = c/d$ AND $e/b \neq f/d$, THEN THE GRAPHS HAVE THE SAME SLOPE BUT DIFFERENT y-INTERCEPTS. THEY ARE PARALLEL LINES, AND THERE IS NO SOLUTION.

3. IF $a/b \neq c/d$, THEN THE EQUATIONS HAVE DIFFERENT SLOPES. THEIR GRAPHS CROSS AT A SINGLE POINT, WHICH SOLVES BOTH EQUATIONS SIMULTANEOUSLY.



Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.

Horizontal and Vertical Lines

IN THE EQUATION

$ax + by = c$

WE KEEP ASSUMING $b \neq 0$. THIS MEANS WE'VE LOOKED ONLY AT EQUATIONS LIKE

$2x + 6y = 4$

$9x - 503y = 7{,}021{,}077$

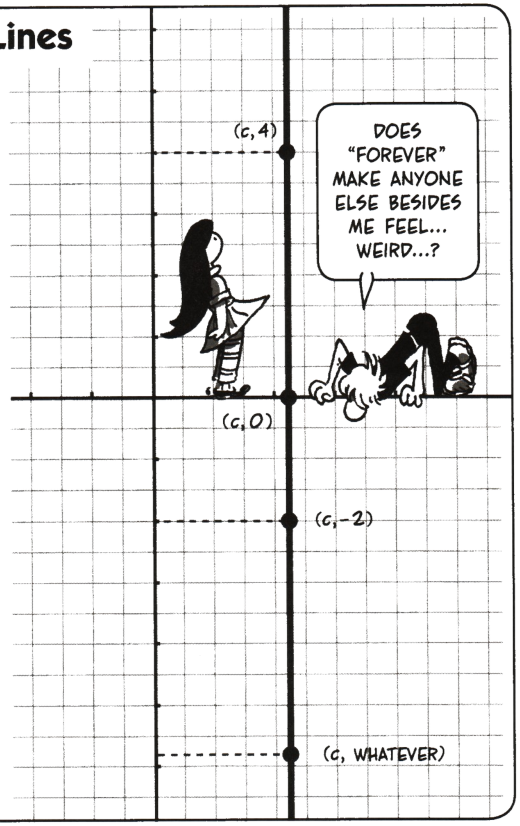
OR EVEN

$y = 8$

(a, THE COEFFICIENT OF x, CAN BE ZERO!) BUT WHAT ABOUT WHEN $b = 0$? THESE ARE EQUATIONS LIKE

$$x = c$$

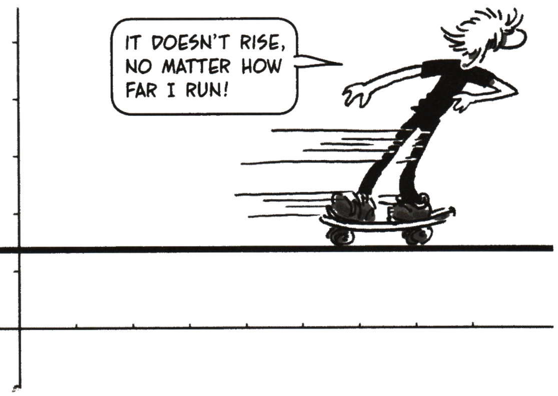
WHICH HAS A **VERTICAL GRAPH.** IT'S ALL THE POINTS WITH AN x-COORDINATE EQUAL TO c. A VERTICAL LINE'S SLOPE IS... **INFINITE.** IT RISES (AND FALLS) FOREVER WITH NO RUN AT ALL.



WHEN $a = 0$, ON THE OTHER HAND, THE EQUATION LOOKS LIKE

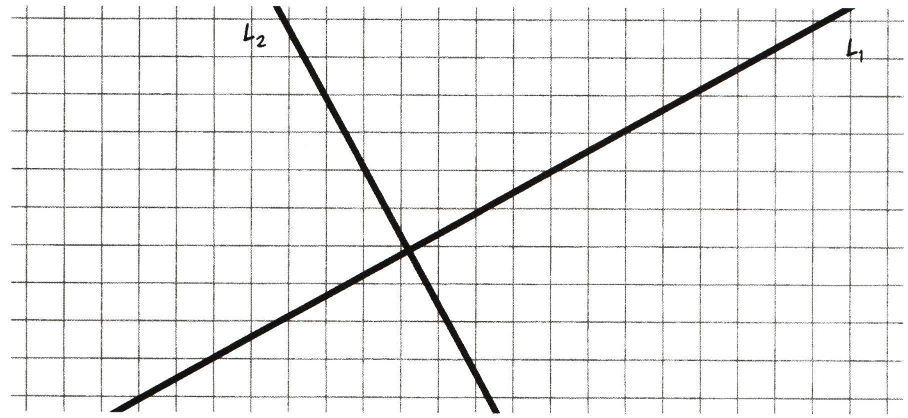
$$y = c$$

ITS GRAPH IS HORIZONTAL, A LINE OF SLOPE **ZERO** (NO RISE, ALL RUN).

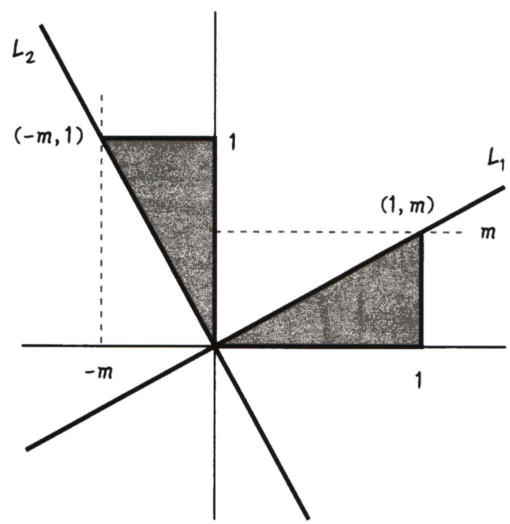


Perpendicular Lines

FINALLY, JUST FOR FUN, LET'S SEE WHAT IT MEANS ALGEBRAICALLY FOR TWO LINES TO MEET AT A RIGHT ANGLE. SUCH LINES ARE CALLED **PERPENDICULAR.** AS YOU GO AROUND THEIR INTERSECTION, ALL FOUR ANGLES ARE EQUAL, LIKE THE CORNERS OF A SQUARE. THE COORDINATE AXES ARE AN EXAMPLE. GIVEN TWO PERPENDICULAR LINES L_1 AND L_2, LET'S SUPPOSE THAT L_1 HAS SLOPE m. WHAT'S THE SLOPE OF L_2 IN TERMS OF m?



FIRST, SLIDE THE TWO LINES SO THAT THEY CROSS AT THE ORIGIN. THE SLOPES REMAIN THE SAME, AND THE LINE L_1 NOW CONTAINS THE POINT $(1, m)$. (THE LINE $y = mx$ ALWAYS CONTAINS THE POINT $(1, m)$!)



IN THE DIAGRAM, THE TWO GRAY TRIANGLES ARE EXACTLY THE SAME, ONLY TURNED. EACH HAS A SIDE OF LENGTH 1 AND A SIDE OF LENGTH m. SO L_2 CONTAINS THE POINT $(-m, 1)$.

THIS MEANS THAT L_2 HAS SLOPE

$$\frac{1}{-m} = -\frac{1}{m}$$

PERPENDICULAR LINES HAVE SLOPES THAT ARE EACH OTHER'S **NEGATIVE RECIPROCAL** (AS LONG AS NEITHER LINE IS VERTICAL!).

Excerpts from Gonick, Larry (2015). *The Cartoon Guide to Algebra*. New York, NY. Harper Collins.