

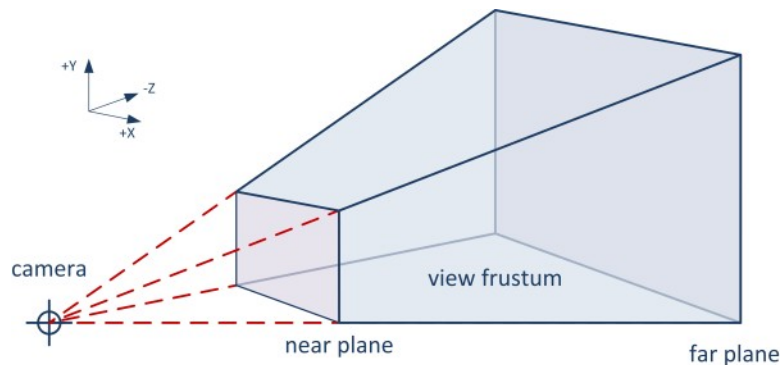
Our only sources of information on the mathematics of ancient Egypt are the Moscow Mathematical Papyrus and the Rhind Mathematical Papyrus. The Moscow Mathematical Papyrus dates from 1850 B.C., about the time of Abraham. V. S. Golenishchev acquired it in 1893 and brought it to Moscow.

The most interesting problem in the Moscow Mathematical Papyrus is Problem 14. This is a computation of the volume of a frustum, using the correct formula. A frustum is a pyramid with a similar pyramid cut off its top. If it has a square base of side a , and a square top of side b , and if its height is h , then, as the ancient Egyptians realised, the volume of the frustum is

$$\frac{h(a^2 + ab + b^2)}{3}$$

Note that if $b = 0$, we get the formula for the volume of a square-base pyramid: $a^2h/3$.

We do not know how the Egyptians arrived at these formulas. Perhaps it was by trial and error.



Attempts have been made to use the dimensions of the Great Pyramid (built about 2600 B.C.) to draw conclusions about Egyptian mathematics. For example, it is claimed that half the perimeter of the base of the pyramid, divided by its height, equals 3.14. From this it is supposed to follow that the Egyptians of 2600 B.C. knew the value of π to two decimal places. Against this idle speculation, we advance the following considerations. (1) Over the centuries, people have taken stone from the pyramid for their own building projects; the original surface of the pyramid has thus disappeared, and we have no way of knowing its original dimensions with two-decimal-place accuracy. (2) There are dozens of ratios one can calculate given the alleged dimensions of a pyramid; it is not surprising if one of them happens to be close to π . (3) In the Rhind Papyrus, the value used for π is about 3.16; if the Egyptians knew a better approximation for π in 2600 B.C., they would not have been using a worse one in 1650 B.C.²

The Sumerians lived in the southern part of Mesopotamia (Iraq). About 2000 B.C., their civilisation was absorbed by the Babylonians, and Babylonian culture reached its peak about 575 B.C., under Nebuchadnezzar. The mathematical achievements we shall discuss in this chapter are recorded on the clay tablets of the Sumerians and Babylonians. Most of these achievements go back as far as 2000 B.C. — about the time when Abraham's father was living in the Sumerian city of Ur. We shall use the word 'Babylonian' for what is perhaps more accurately described as 'Mesopotamian' mathematics.

The Babylonians used a counting scale, not of 10, but of 60, and this scale was taken over into Greek astronomy by Hipparchus of Nicaea (about 150 B.C.). It is thanks to the Babylonians, and Hipparchus, that we have 60 minutes in an hour. According to the prophet Ezekiel (573 B.C.), in the ancient system of weights, scale 60 was endorsed by God himself:

Lord Yahweh says this: ... Twenty shekels, twenty-five shekels and fifteen shekels are to make one mina (Ezekiel 45:9-12).

The Babylonians could solve linear and quadratic equations. They could even solve the simultaneous equations

$$\begin{aligned}x^8 + x^6 y^2 &= (3,200,000)^2 \\ xy &= 1,200\end{aligned}$$

The Babylonians built pyramid-shaped 'ziggurats'. The first story of a ziggurat might measure $n \times n \times 1$, the second story $(n-1) \times (n-1) \times 1$, and so on — with the top two stories measuring $2 \times 2 \times 1$ and $1 \times 1 \times 1$. The volume of such a ziggurat is

$$1^2 + 2^2 + \cdots + (n-1)^2 + n^2$$

and the Babylonians knew that this equals

$$\frac{n(n+1)(2n+1)}{6}$$

a result first proved by Archimedes (287–212 B.C.).

The Bible tells us that there was once an attempt to build a ziggurat 'with its top reaching heaven' (Genesis 11:4). Perhaps the promoters of the Tower of Babel mistakenly believed that the infinite series $1^2 + 2^2 + 3^2 + \cdots$ converges.

The Babylonians knew the formulas for the areas of the triangle, trapezium, and circle. According to a clay tablet found in Susa in 1936, they used the value $3\frac{1}{8}$ for π .

