33

The ancient Greek world was not confined to what we now call 'Greece', but extended to Ionia (west Turkey) in the east and to southern Italy in the west. The first Greek mathematician and philosopher was Thales of Miletus, a contemporary of the prophet Ezekiel (600 B.C.). (Miletus was on the southwestern coast of Turkey.) According to Proclus, Thales visited Egypt and learned geometry there. Thales predicted the solar eclipse that occurred over Greece and Mesopotamia on May 28, 585 BC.

Plato repeats a story about Thales being an absent-minded professor who was so preoccupied with celestial matters that he failed to observe what was in front of his feet and once fell into a well (*Theaetetus* 174a). According to other anecdotes, however, Thales had a practical mind. He constructed an almanac, figured out how to calculate the distance of ships from shore, and he once cornered the market in olive oil.

Thales is associated with a number of theorems in geometry:

(1) a circle is bisected by a diameter;
(2) the base angles of an isosceles triangle are equal;
(3) vertically opposite angles are equal;
(4) two triangles are congruent if their angles and one side are equal;
(5) an angle in a semicircle is right.

Theorem (5) is *Thales's theorem*. What it means is that if AC is a diameter of a circle and B is a point on the circumference of the circle (other than A or C), then $\angle ABC$ is a right angle (has 90 degrees).

All these theorems were known by the Egyptians and Mesopotamians. The reason they are associated with Thales is that he was the first person to offer *proofs* for them. This was an essential difference between pre-Greek and Greek mathematics: the Greeks established the logical connections among their results, deducing the theorems from a small set of starting assumptions or *axioms*.

As a philosopher, Thales is famous for his statement that everything is made of water. This statement committed Thales to the following views:

(a) there is more than one thing;
(b) there is only one *kind* of thing (namely, water);
(c) the physical universe should not be understood in terms of unconnected fragments (e.g., quarks), but in terms of a continuous substance (e.g., space).

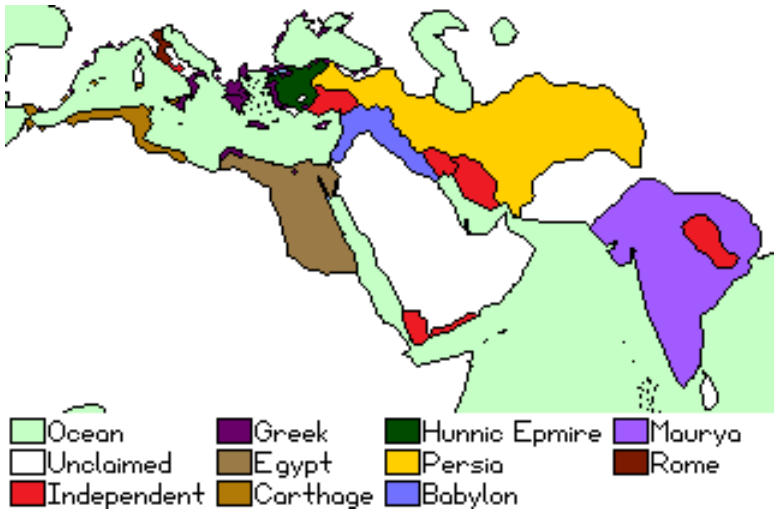
The fact that contemporary physicists disagree with Thales about (b) and (c) is less important than the fact that it was Thales who first raised these issues.

34

Pythagoras (570–500 B.C.) was born in Samos, a Greek island off the coast of what is now Turkey. According to Iamblichus, Porphyry, and Diogenes Laertius, Pythagoras studied under the Babylonians, and he may have met the prophet Daniel in Babylon. From the clay tablet Plimpton 322, we know that the Babylonians had a well-worked-out theory of 'Pythagorean triangles', and Pythagoras would have learnt this from them. Pythagoras may have discovered the first proof of the 'theorem of Pythagoras', but he certainly did not discover the theorem itself.

According to Iamblichus, Porphyry, and Diogenes Laertius, Pythagoras also studied under the 'Magi', or Zoroastrians. Indeed, it is not impossible that Pythagoras talked with Zoroaster himself. Nor is it impossible that Pythagoras studied in India. His belief in reincarnation certainly had an Indian origin. Perhaps Pythagoras met Buddha, another of his contemporaries.

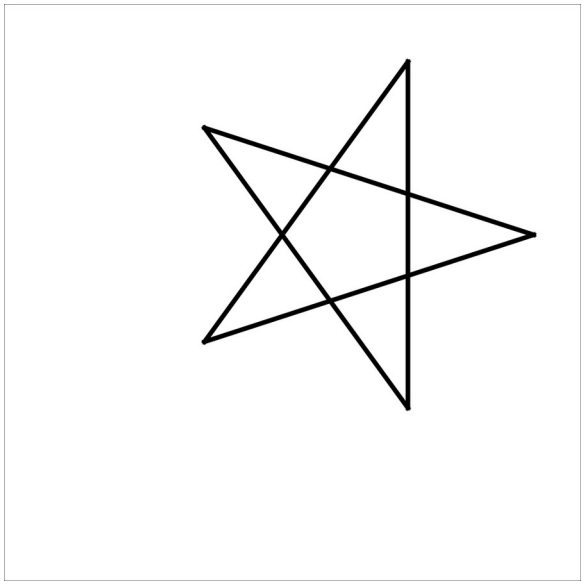
About 525 B.C., Pythagoras moved to Croton, a town in southern Italy, and founded the brotherhood of the Pythagoreans. He married a woman Pythagorean called Theano. Theano may have been the first woman mathematician.



Excerpts from Anglin, W.S. (1994). *Mathematics: A Concise History*. New York, NY. Springer-Verlag.

Whereas Thales had claimed that 'all is water', Pythagoras taught that 'all is number'. For Pythagoras, this implied that everything could be understood in terms of whole numbers and their ratios. In particular, every line segment was a whole number or ratio of whole numbers. Although the discovery of the irrationality of the diagonal of the square of side 1 was made by followers of Pythagoras, Pythagoras himself was not aware of this.

Pythagoras gave a special place to the number 10. He called it the 'divine number'. He was attracted by it probably for the following reasons. It is the scale in which the ancient Greeks counted. As the sum of the first four positive integers, it represents the three dimensions — with 1 for points, 2 for lines, 3 for planes, and 4 for solids. Finally, there are ten vertices in the five-pointed Pythagorean star.



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The Pythagoreans ascribed all their mathematical discoveries to Pythagoras, but there is not, in fact, a single theorem we can safely credit to the master. Pythagorean accomplishments include the following.

(1) A Proof of the Theorem of Pythagoras

The Pythagoreans were responsible for the proof of this theorem found in Euclid. They also found a proof of the converse of this theorem.

(2) Means

The Pythagoreans examined the arithmetic mean $(a+b)/2$, the geometric mean $\sqrt{ab}$, the harmonic mean $2ab/(a+b)$, and the relationships among them.

(3) Perfect and Amicable Numbers

A *perfect number* is a positive integer, such as 6, which equals the sum of its proper divisors: $6 = 1+2+3$. The Pythagoreans found a formula giving even perfect numbers. See Chapter 5. An *amicable pair* is two positive integers, each of which is the sum of the proper divisors of the other. Iamblichus (300 A.D.) credits Pythagoras with a knowledge of the amicable pair 220 and 284.

(4) Regular Solids

The Pythagoreans discovered the dodecahedron, and proved that there are just 5 regular polyhedra. This accomplishment was unsurpassed until J. Kepler (1571–1630) discovered the lesser and greater stellated dodecahedra. See Chapter 6.

(5) The Irrationality of $\sqrt{2}$

The Pythagoreans discovered that $\sqrt{2}$ is not a ratio of whole numbers. They used integer solutions of $x^2 - 2y^2 = 1$ to find good approximations to it. See Chapter 7.