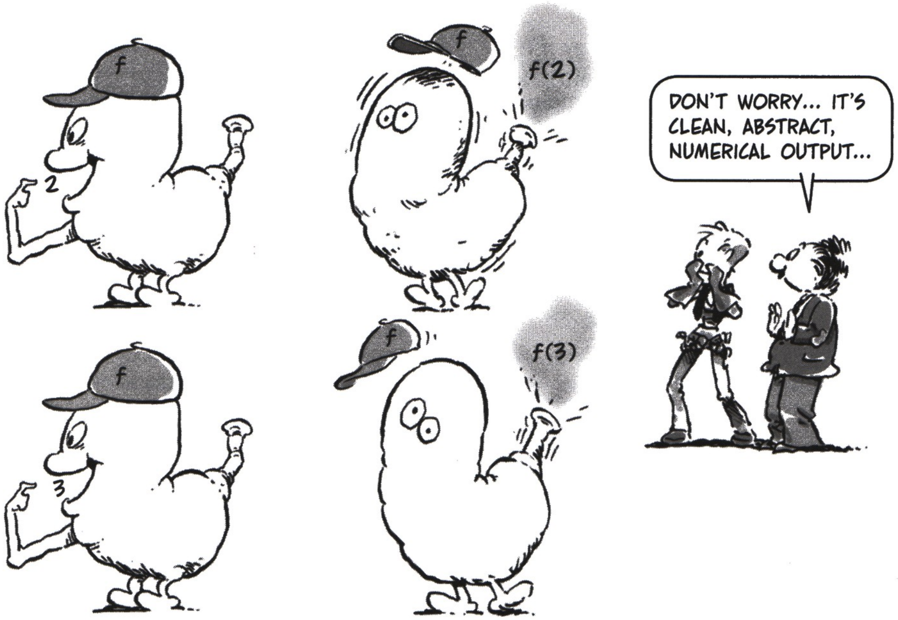
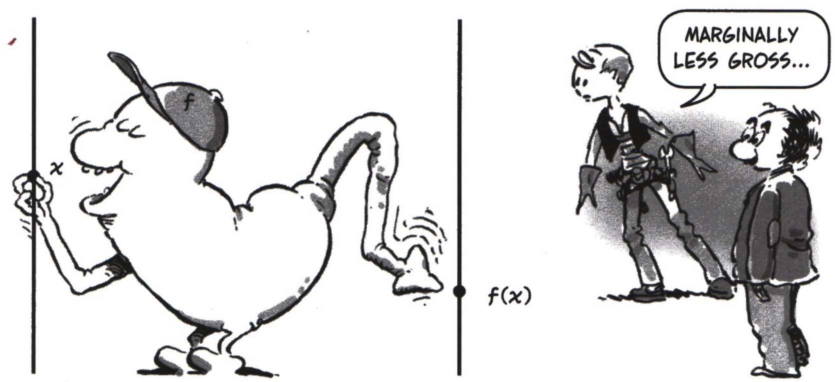
1

A FUNCTION IS A SORT OF **INPUT-OUTPUT DEVICE** OR **NUMBER-PROCESSOR.** A FUNCTION (CALL IT f) EATS AND SPEWS NUMBERS IN A SPECIFIC WAY. FOR EACH NUMBER EATEN (CALL IT x), f OUTPUTS A SINGLE, UNIQUE NUMBER, $f(x)$, PRONOUNCED "EFF OF ECKS." f IS LIKE A RULE THAT TRANSFORMS x INTO $f(x)$. IN GOES x, OUT COMES $f(x)$.

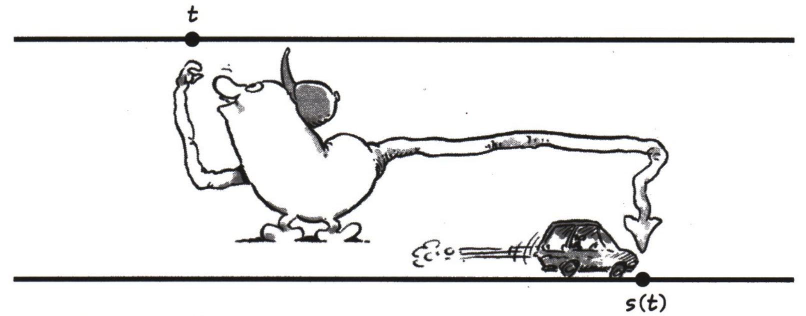


IF YOU DON'T LIKE YOUR OUTPUT FLOATING AROUND IN THE AIR LIKE SWAMP GAS, THEN THINK OF NUMBERS AS LYING ALONG A LINE. IN THAT CASE, YOU CAN IMAGINE A FUNCTION f EATING NUMBERS FROM ONE LINE AND MERELY **POINTING** TO THE CORRESPONDING OUTPUT VALUES ON THE OTHER LINE.

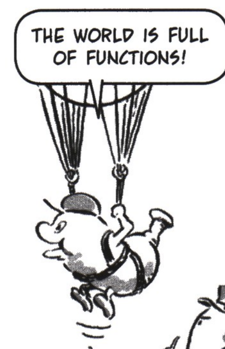


2

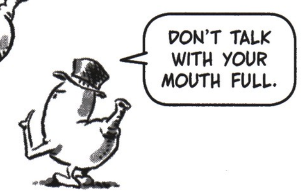
FOR EXAMPLE, A CAR'S POSITION s IS A FUNCTION OF TIME t. YOU CAN THINK OF s AS READING TIME (OR EATING IT AS INPUT!) FROM A TIMELINE AND POINTING TO THE CAR'S POSITION $s(t)$ ON THE TRACK.



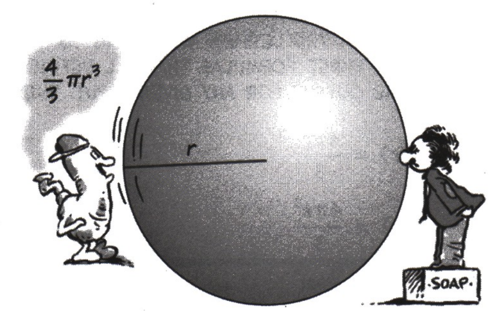
More Examples:



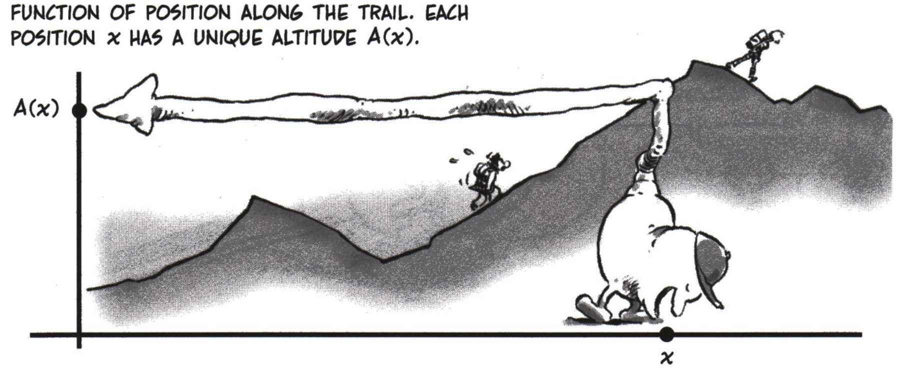
ATMOSPHERIC PRESSURE DEPENDS ON ALTITUDE: AT EACH ALTITUDE A, THERE IS A DEFINITE PRESSURE $P(A)$. THE FUNCTION P EATS ALTITUDE AND OUTPUTS PRESSURE.



AS A SPHERICAL BALLOON INFLATES, ITS VOLUME IS A FUNCTION OF THE RADIUS. EACH RADIUS r DETERMINES A UNIQUE VOLUME $V(r)$.



ON A STRAIGHT MOUNTAIN TRAIL, ALTITUDE IS A FUNCTION OF POSITION ALONG THE TRAIL. EACH POSITION x HAS A UNIQUE ALTITUDE $A(x)$.



Excerpts from Gonick, Larry (2012). *The Cartoon Guide to Calculus*. New York, NY. Harper Collins.

3

IN THE EXAMPLE OF THE SPHERICAL BALLOON, THE VOLUME FUNCTION V WAS CALCULATED FROM THE RADIUS r BY MEANS OF A **FORMULA:**

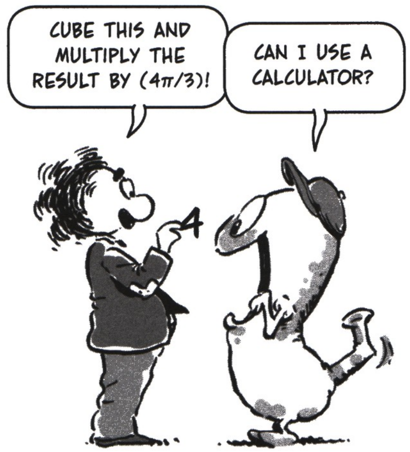
$$V(r) = \frac{4\pi r^3}{3}$$

TO FIND THE VOLUME ASSOCIATED WITH A PARTICULAR RADIUS, SAY $r = 10$, WE INPUT, OR **PLUG IN,** THAT NUMBER IN PLACE OF r:

$$V(10) = \frac{4\pi(10)^3}{3} = \frac{4000}{3}\pi$$

$$\approx 4{,}188.79\ldots$$

(THE SIGN "≈" MEANS "IS APPROXIMATELY EQUAL TO.")

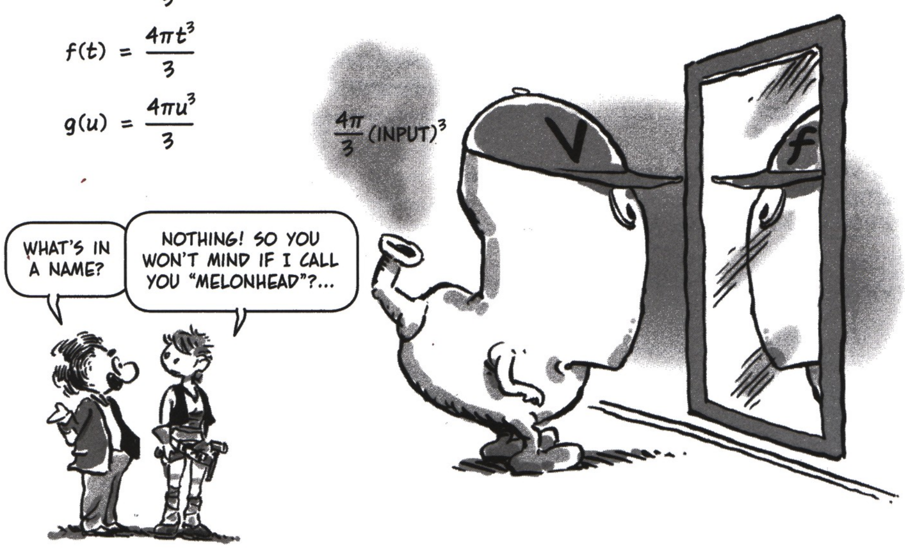


IMPORTANT: THE LETTERS WE ASSIGN TO THE FUNCTION AND VARIABLE DON'T MATTER! HERE ARE THREE FORMULAS THAT ALL DEFINE THE SAME FUNCTION BECAUSE THEY PRODUCE THE SAME OUTPUT FOR ANY GIVEN INPUT. THEY ALL DESCRIBE THE SAME RULE.

$$V(r) = \frac{4\pi r^3}{3}$$

$$f(t) = \frac{4\pi t^3}{3}$$

$$g(u) = \frac{4\pi u^3}{3}$$



4

HERE IS A SLIGHTLY MORE COMPLICATED EXAMPLE. SUPPOSE h IS GIVEN BY THIS FORMULA:

$$h(x) = \sqrt{x^2 - 1}$$

WE COMPUTE A FEW VALUES...

$$h(1) = \sqrt{1^2 - 1} = 0$$

$$h(2) = \sqrt{2^2 - 1} = \sqrt{3}$$

$$h(\sqrt{5}) = \sqrt{5 - 1} = 2$$

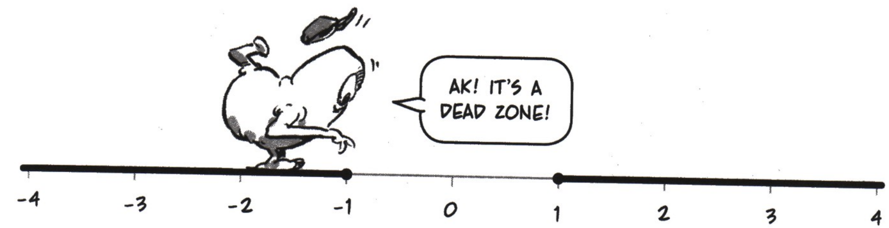
ETC...

AND COMPILE A LITTLE TABLE. IT'S FULL OF GAPS, BUT YOU CAN FILL IN MANY MISSING VALUES... EXCEPT...

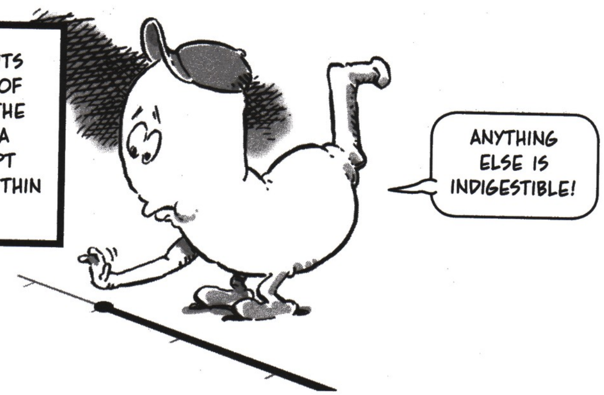
x	$h(x)$
-3	$\sqrt{8}$
-2.9	$\sqrt{7.41}$
-2.8	$\sqrt{6.84}$
-2	$\sqrt{3}$
-1	0
1	0
2	$\sqrt{3}$
$\sqrt{5}$	2
3	$\sqrt{8}$

ETC...

WHEN x IS BETWEEN -1 AND 1, THE EXPRESSION INSIDE THE SQUARE ROOT SIGN IS NEGATIVE: $x^2 - 1 < 0$. IN THAT CASE, $h(x)$ IS **UNDEFINED,** BECAUSE NEGATIVE NUMBERS HAVE NO (REAL) SQUARE ROOT. EVERY INPUT ACCEPTED BY h MUST HAVE A VALUE EITHER ≥ 1 OR ≤ -1. NOTHING ELSE IS ALLOWED!



GIVEN ANY FUNCTION, ITS **DOMAIN** IS THE SET OF ALL NUMBERS WHERE THE FUNCTION IS DEFINED. A FUNCTION f WILL ACCEPT INPUTS **ONLY** FROM WITHIN ITS DOMAIN.



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