WE USUALLY DESCRIBE A FUNCTION'S DOMAIN IN TERMS OF **INTERVALS** OF NUMBERS. GIVEN ANY TWO NUMBERS a AND b, WITH $a<b$, WE USE THIS NOTATION:

(a, b), THE **OPEN** INTERVAL BETWEEN a AND b, MEANS ALL THE NUMBERS LYING BETWEEN a AND b **EXCLUDING** THE ENDPOINTS a AND b.

(a, b) IS ALL x WITH $a<x<b$

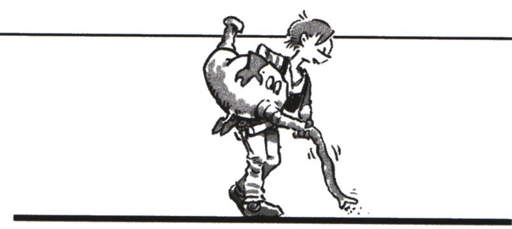
$[a, b]$, THE **CLOSED** INTERVAL BETWEEN a AND b, MEANS ALL THE NUMBERS LYING BETWEEN a AND b **INCLUDING** THE ENDPOINTS.

$[a, b]$ IS ALL x WITH $a \le x \le b$

AN "**INFINITE** INTERVAL" REFERS TO ALL THE NUMBERS GREATER THAN SOME NUMBER c. WE WRITE THIS AS $[c, \infty)$ IF c IS INCLUDED AND (c, ∞), IF NOT. SIMILARLY, ON THE LEFT ARE $(-\infty, d]$ AND $(-\infty, d)$. THE INFINITY SIGN ∞ DOES NOT REPRESENT ANY NUMBER; IT'S SIMPLY A CONVENIENCE TO BE USED IN SITUATIONS LIKE THIS. IT IS NEVER INCLUDED IN ANY INTERVAL, BECAUSE IS ISN'T A NUMBER!

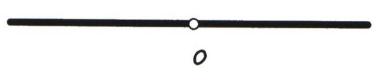
$(-\infty, d)$ IS ALL x WITH $x < d$

$[c, \infty)$ IS ALL x WITH $c \le x$



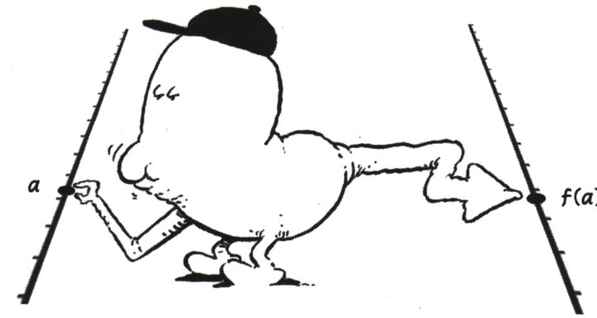
IN TERMS OF INTERVALS, THEN, THE DOMAIN OF $h(x) = \sqrt{x^2 - 1}$ IS EVERYTHING **OUTSIDE** THE INTERVAL $(-1, 1)$.

THE DOMAIN OF $g(x) = \frac{1}{x}$ IS ALL $x \neq 0$. (DIVIDING BY 0 IS FORBIDDEN.)

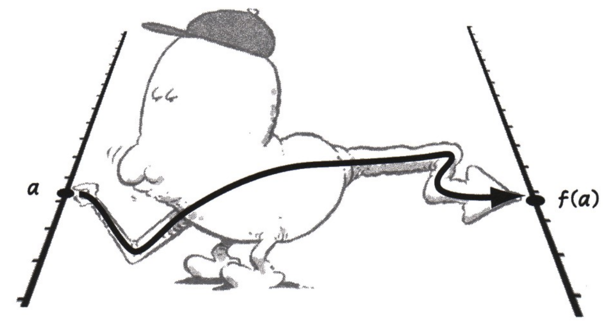


THE DOMAIN OF $P(x) = x^2 + 3$ IS ALL REAL NUMBERS WITHOUT RESTRICTION.

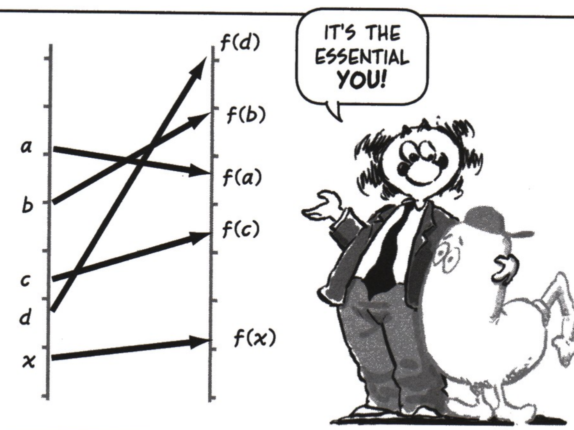
NOW RETURN TO OUR IMAGE OF A FUNCTION PICKING UP INPUTS FROM ONE NUMBER LINE AND POINTING TO OUTPUTS ON ANOTHER NUMBER LINE.



IF WE LIKE, WE CAN LET THE FUNCTION'S CARTOON BODY FADE AWAY AND CONCENTRATE ON THE ACT OF **POINTING.**



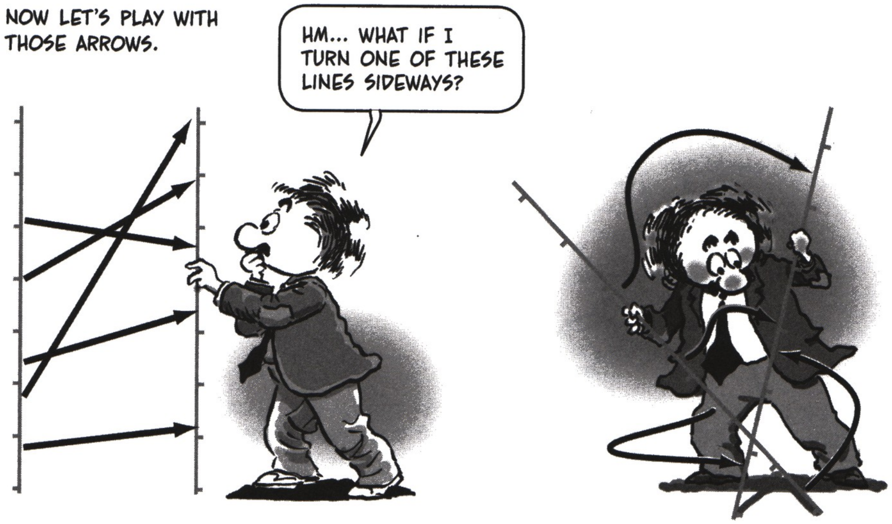
IN THIS VIEW, A FUNCTION IS SIMPLY A **COLLECTION OF ARROWS** POINTING FROM ONE NUMBER LINE TO ANOTHER. A SINGLE ARROW EMERGES FROM EACH x IN THE DOMAIN OF f AND POINTS TO THE VALUE $f(x)$.



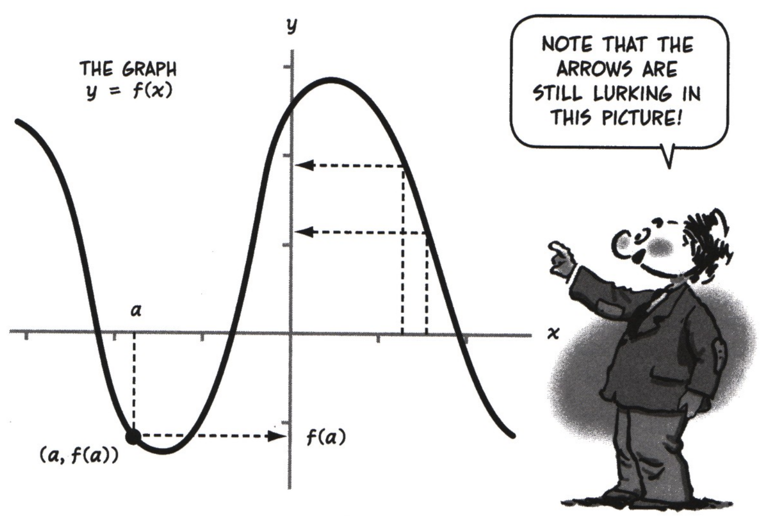
Excerpts from Gonick, Larry (2012). *The Cartoon Guide to Calculus*. New York, NY. Harper Collins.

7

NOW LET'S PLAY WITH THOSE ARROWS.



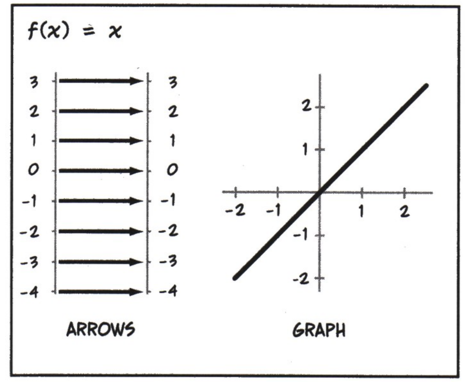
WHEN THE FIRST LINE, OR **AXIS**, IS TURNED SIDEWAYS, WE CAN VIEW A FUNCTION AS A **GRAPH**. THE INPUTS x ARE ON THE HORIZONTAL AXIS, THE OUTPUTS y ARE ON THE VERTICAL AXIS, AND ABOVE (OR BELOW) ANY POINT a ON THE x-AXIS WE PLOT A POINT $(a, f(a))$, WITH y-COORDINATE EQUAL TO THE VALUE OF THE FUNCTION f AT a.

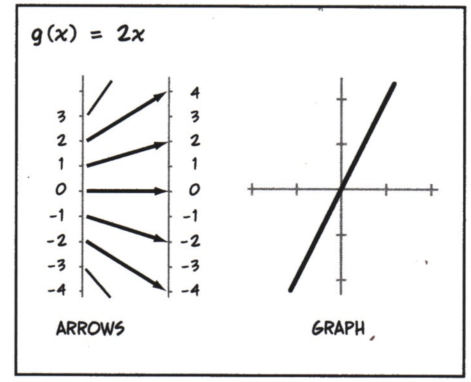


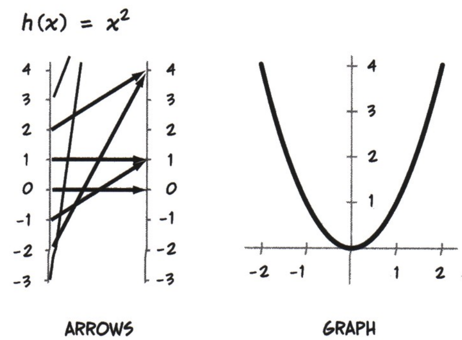
THE CURVE CONSISTS OF ALL POINTS (x, y) WITH $y = f(x)$, A PHRASE WE ABBREVIATE BY SAYING "THE **GRAPH** $y = f(x)$."

8

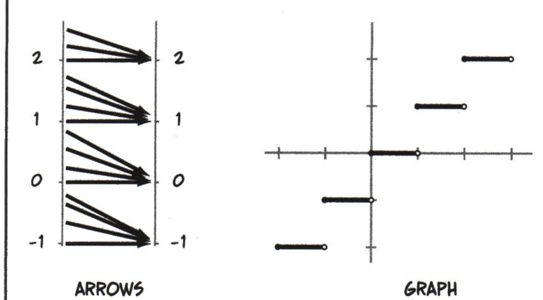
HERE ARE SOME SIMPLE EXAMPLES.



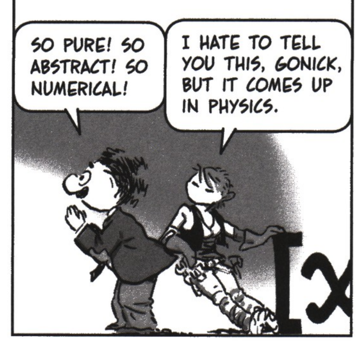




$F(x)$ = THE LARGEST INTEGER (WHOLE NUMBER) $\leq x$, SOMETIMES WRITTEN $[x]$. (SO $[5] = 5$, $[5.7] = 5$, $[-1.6] = -2$, $[-0.3] = -1$, ETC.



THIS LAST WAS AN EXAMPLE OF A FUNCTION THAT HAS NO FORMULA IN THE USUAL SENSE.



Excerpts from Gonick, Larry (2012). *The Cartoon Guide to Calculus*. New York, NY. Harper Collins.