All of the transformations we have discussed have maintained the shape and size of the figure. However, a dilation changes the size of the figure. The dilated figure is similar to the original figure. Dilations using the origin as a center of projection can be achieved by multiplying the vertex matrix by the scale factor needed for the dilation. *All dilations in this lesson are with respect to the origin.*

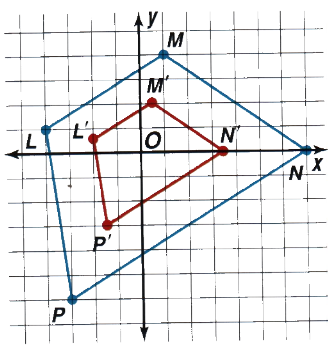
4 **A trapezoid has vertices at $L(-4, 1)$, $M(1, 4)$, $N(7, 0)$, and $P(-3, -6)$. Find the coordinates of the dilated trapezoid $L'M'N'P'$ for a scale factor of 0.5. Describe the dilation.**

First write the coordinates of the vertices as a matrix. Then do a scalar multiplication using the scale factor.

$$0.5\begin{bmatrix} -4 & 1 & 7 & -3 \\ 1 & 4 & 0 & -6 \end{bmatrix} = \begin{bmatrix} -2 & 0.5 & 3.5 & -1.5 \\ 0.5 & 2 & 0 & -3 \end{bmatrix}$$

The vertices of the image are $L'(-2, 0.5)$, $M'(0.5, 2)$, $N'(3.5, 0)$, and $P'(-1.5, -3)$.

The image has sides that are half the length of the original figure.



Real World Application

MILITARY SCIENCE

When the U.S. Army needs to determine how many soldiers or officers to put in the field, they turn to mathematics. A system called the *Manpower Long-Range Planning System* (MLRPS) enables the army to meet the personnel needs for 7- to 20-year planning periods. Analysts are able to effectively use the MLRPS to simulate gains, losses, promotions, and reclassifications. This type of planning requires solving up to 9,060 inequalities with 28,730 variables! However, with a computer, a problem like this can be solved in less than five minutes.

The Army's MLRPS uses a procedure called **linear programming.** Many practical applications can be solved by using this method. The nature of these problems is that certain **constraints** exist or are placed upon the variables, and some function of these variables must be maximized or minimized. The constraints are often written as a system of linear inequalities.

The following procedure can be used to solve linear programming applications.

1. Define variables.
2. Write the constraints as a system of inequalities.
3. Graph the system and find the coordinates of the vertices of the polygon formed.
4. Write an expression whose value is to be maximized or minimized.
5. Substitute values from the coordinates of the vertices into the expression.
6. Select the greatest or least result.

In Lesson 2-6, you found the maximum and minimum values for a given function in a defined polygonal convex region. In linear programming, you must use your reasoning abilities to determine the function to be maximized or minimized and the constraints that form the region.

1 MANUFACTURING **Suppose a lumber mill can turn out 600 units of product each week. To meet the needs of its regular customers, the mill must produce 150 units of lumber and 225 units of plywood. If the profit for each unit of lumber is \$30 and the profit for each unit of plywood is \$45, how many units of each type of wood product should the mill produce to maximize profit?**

Define variables. Let x = the units of lumber produced.
Let y = the units of plywood produced.

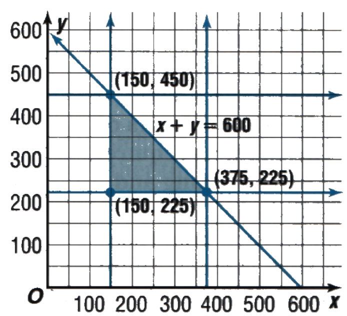
Write inequalities.

$x \geq 150$ — *There cannot be less than 150 units of lumber produced.*

$y \geq 225$ — *There cannot be less than 225 units of plywood produced.*

$x + y \leq 600$ — *The maximum number of units produced is 600.*

Graph the system.



The vertices are at (150, 225), (375, 225), and (150, 450).

Write an expression. Since profit is \$30 per unit of lumber and \$45 per unit of plywood, the profit function is $P(x, y) = 30x + 45y$.

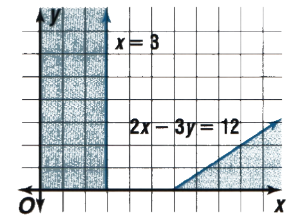
Substitute values.

$P(150, 225) = 30(150) + 45(225)$ or 14,625
$P(375, 225) = 30(375) + 45(225)$ or 21,375
$P(150, 450) = 30(150) + 45(450)$ or 24,750

Answer the problem. The maximum profit occurs when 150 units of lumber are produced and 450 units of plywood are produced.

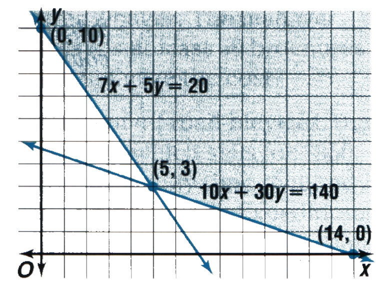
In certain circumstances, the use of linear programming is not helpful because a polygonal convex set is not defined. Consider the graph at the right, based on the following constraints.

$x \geq 0$
$y \geq 0$
$x \leq 3$
$2x - 3y \geq 12$



The constraints do not define a region with any points in common in Quadrant I. When the constraints of a linear programming application cannot be satisfied simultaneously, then the problem is said to be **infeasible**.

Sometimes the region formed by the inequalities in a linear programming application is **unbounded.** In that case, an optimal solution for the problem may not exist. Consider the graph at the right. A function like $f(x, y) = x + 2y$ has a *minimum* value at (5, 3), but it is not possible to find a maximum value.



It is also possible for a linear programming application to have two or more optimal solutions. When this occurs, the problem is said to have **alternate optimal solutions.** This usually occurs when the graph of the function to be maximized or minimized is parallel to one side of the polygonal convex set.

Excerpts from Holliday, B. Cuevas, G. McClure, M. Carter, J. Marks, D. (2007). *Advanced Mathematical Concepts*. New York, NY. McGraw Hill Glencoe.