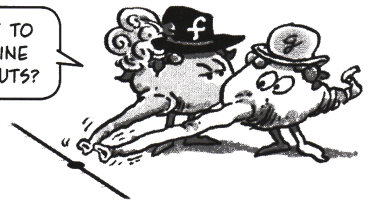
Add, Multiply, Divide

FUNCTIONS CAN BE COMBINED IN VARIOUS WAYS, JUST AS NUMBERS CAN. IF f AND g HAVE OVERLAPPING DOMAINS, WE CAN ADD, MULTIPLY, AND DIVIDE THE FUNCTIONS WHEREVER THEY ARE BOTH DEFINED. THIS PRODUCES NEW FUNCTIONS $f + g$, fg, AND f/g (AS LONG AS WE'RE CAREFUL NEVER TO DIVIDE BY ZERO).

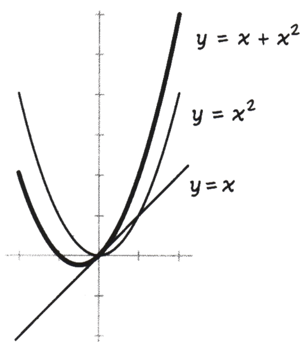
$(f + g)(x) = f(x) + g(x)$

$(fg)(x) = f(x)g(x)$

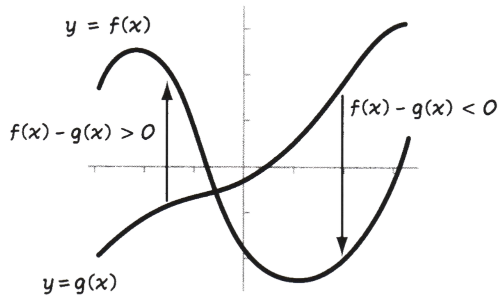
$(f/g)(x) = f(x)/g(x)$ EXCEPT WHERE $g(x) = 0$.



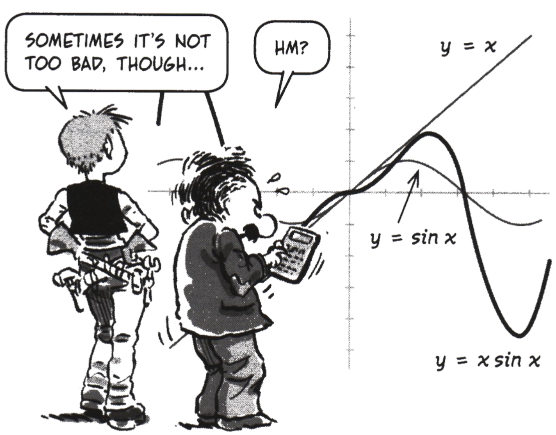
THE GRAPH OF $f + g$ CAN BE BUILT **FROM** THE GRAPHS OF f AND g BY **ADDING** THE y-COORDINATES AT EACH **POINT** x IN THEIR COMMON DOMAIN.



THE DIFFERENCE BETWEEN TWO FUNCTIONS CAN BE VISUALIZED AS THE DIRECTED DISTANCE BETWEEN THEIR GRAPHS.

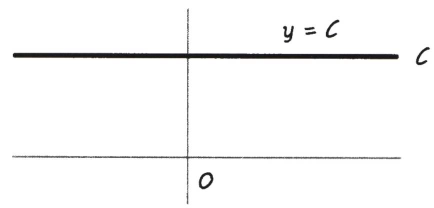


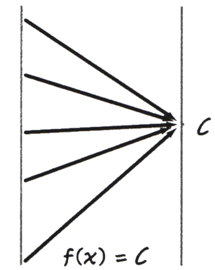
IN GENERAL, THE GRAPHS OF PRODUCTS fg AND QUOTIENTS f/g ARE NOT SO EASILY SEEN IN TERMS OF f AND g. USUALLY THEY MUST BE CALCULATED POINT BY POINT.

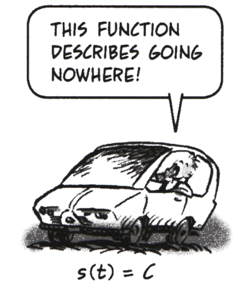


Constants

IF C IS ANY FIXED NUMBER, THEN THERE IS A VERY SIMPLE-MINDED FUNCTION f DEFINED BY $f(x) = C$ FOR ALL x. NOT MUCH OF A FUNCTION, YOU MIGHT SAY, BUT IT IS A FUNCTION! ITS GRAPH IS THE HORIZONTAL LINE $y = C$. ALL ARROWS POINT TO THE SAME NUMBER.

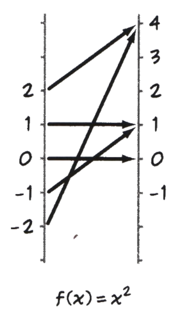


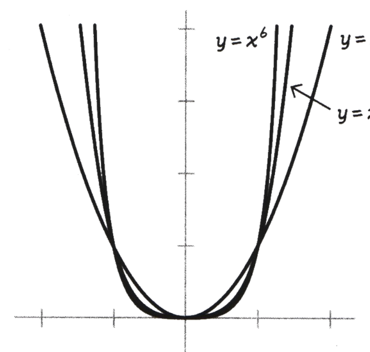


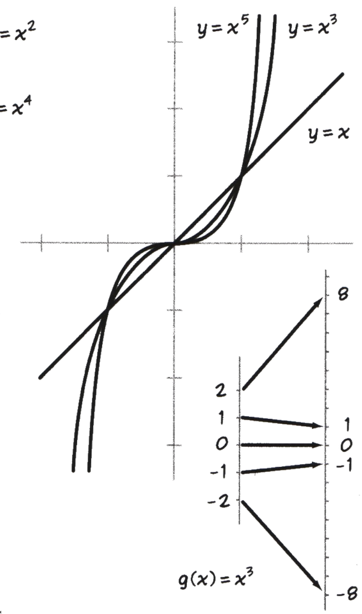


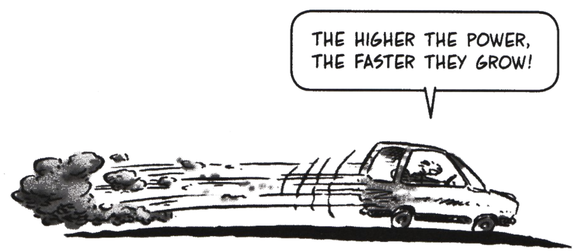
Power Functions

THESE ARE THE FUNCTIONS WITH FORMULA x, x^2, x^3, ..., x^{17}, ... x^n... WHERE n IS A POSITIVE INTEGER. WHEN n IS **EVEN,** THESE FUNCTIONS ALL HAVE BOWL-SHAPED GRAPHS, BECAUSE $(-x)^n = x^n$. POSITIVE AND NEGATIVE INPUTS "LAND" IN THE SAME PLACE. IF n IS **ODD,** THEN $(-x)^n = -(x^n)$, AND THE GRAPHS BEND DOWNWARD ON THE LEFT.









Excerpts from Gonick, Larry (2012). *The Cartoon Guide to Calculus*. New York, NY. Harper Collins.

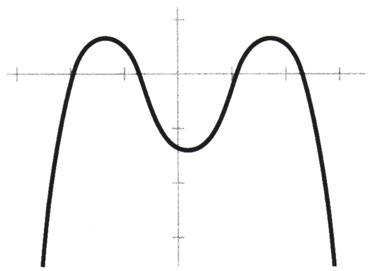
Polynomials

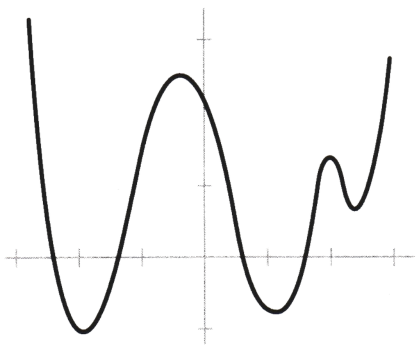
WE ADD CONSTANTS AND MULTIPLES OF POWER FUNCTIONS TO MAKE **POLYNOMIALS,** WHICH HAVE FORMULAS LIKE $2x^2 + x + 41$ OR $x^{15} - x^{14} - 9x$. THE CONSTANT FACTORS ARE CALLED THE POLYNOMIAL'S **COEFFICIENTS,** AND THE LARGEST POWER OF x WITH A NON-ZERO COEFFICIENT IS CALLED THE POLYNOMIAL'S **DEGREE.**

$P(x) = 7x^{10} + 395x^4 + x^3 + 11$ HAS DEGREE 10.

$Q(x) = -x + 9$ HAS DEGREE 1

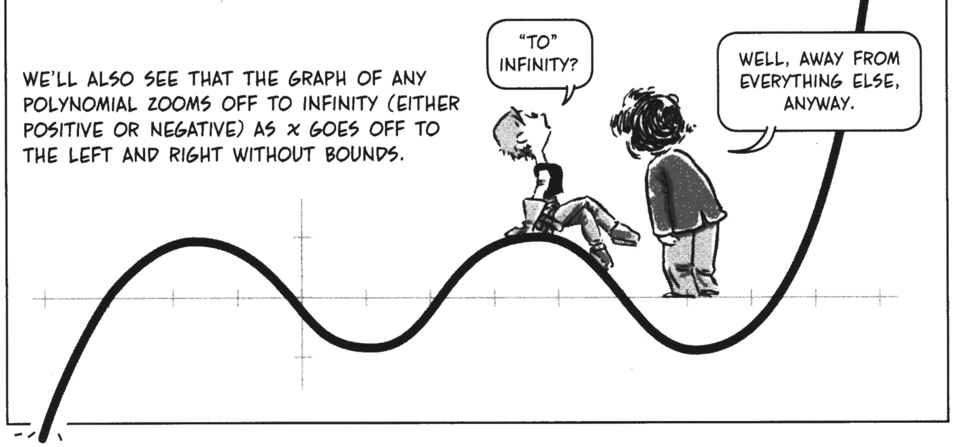
ALGEBRA TEACHES US THAT A POLYNOMIAL P OF DEGREE n HAS NO MORE THAN n **ROOTS,** MEANING NUMBERS $x_1, x_2, \ldots x_m$, WHERE $P(x_i) = 0$.





THIS MEANS THAT THE GRAPH OF AN nTH DEGREE POLYNOMIAL CROSSES THE x-AXIS NO MORE THAN n TIMES. IN FACT, WE WILL SEE THAT THE GRAPH HAS AT MOST $n - 1$ "TURNINGS" WHERE IT CHANGES FROM RISING TO FALLING OR VICE VERSA.

WE'LL ALSO SEE THAT THE GRAPH OF ANY POLYNOMIAL ZOOMS OFF TO INFINITY (EITHER POSITIVE OR NEGATIVE) AS x GOES OFF TO THE LEFT AND RIGHT WITHOUT BOUNDS.



Negative Powers

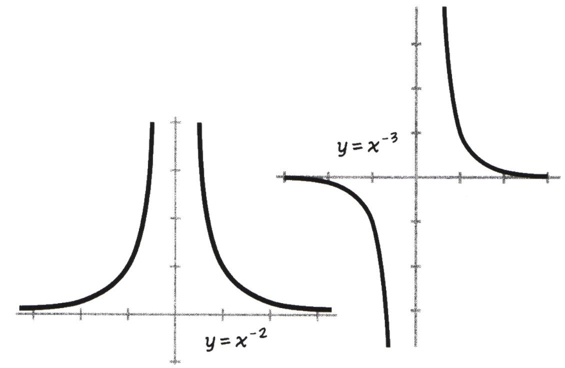
THESE ARE THE FUNCTIONS

$$f(x) = \frac{1}{x^n}, \quad n = 1, 2, 3, \ldots$$

THEY ARE ALSO WRITTEN

$$f(x) = x^{-n}$$

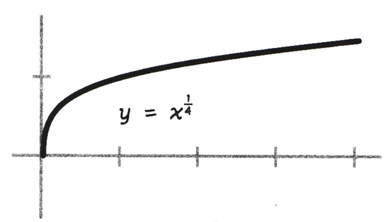
NEGATIVE POWER FUNCTIONS ARE DEFINED FOR ALL $x \neq 0$, AND, LIKE THE POSITIVE POWERS, THEIR GRAPHS DIFFER DEPENDING ON WHETHER n IS ODD OR EVEN.



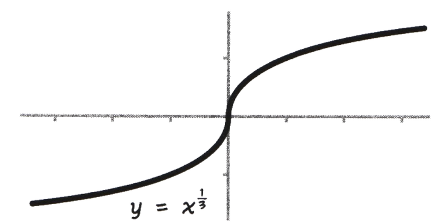
Fractional Powers

IF n IS A POSITIVE INTEGER, $x^{\frac{1}{n}}$ MEANS THE nTH ROOT OF x, $\sqrt[n]{x}$. THE FRACTIONAL NOTATION IS USED TO MAKE THIS FORMULA WORK:

$$(x^{\frac{1}{n}})^n = x^{\frac{1}{n} \cdot n} = x$$

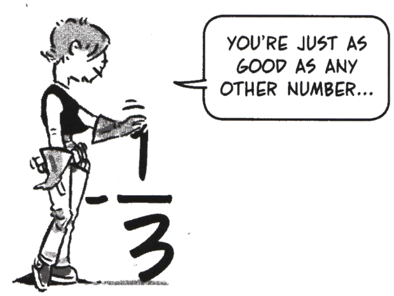


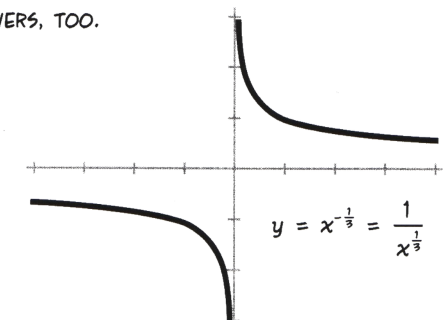
n EVEN: DOMAIN OF $x^{\frac{1}{n}}$ IS ALL $x \geq 0$



n ODD: DOMAIN OF $x^{\frac{1}{n}}$ IS ALL REAL NUMBERS.

THERE CAN BE NEGATIVE FRACTIONAL POWERS, TOO.





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