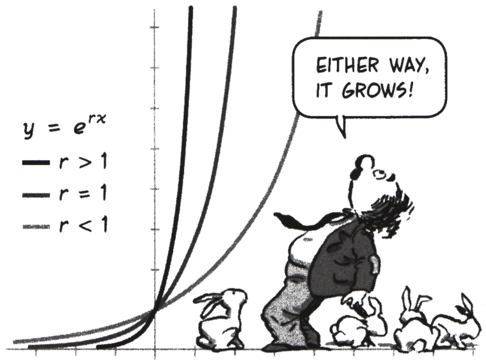
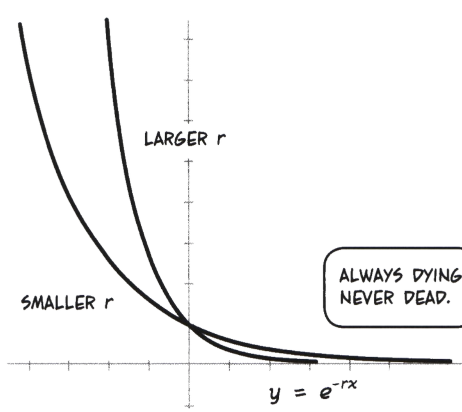
IF r IS ANY POSITIVE NUMBER, THEN THE FUNCTION $h(x) = e^{rx}$ IS AN EXPONENTIAL FUNCTION, BECAUSE

$$e^{rx} = (e^r)^x$$

THE EXPONENTIAL WITH BASE e^r (NOTE THAT $e^r > 1$). IT INCREASES FASTER THAN $exp(x)$ IF $r > 1$ AND SLOWER IF $r < 1$.



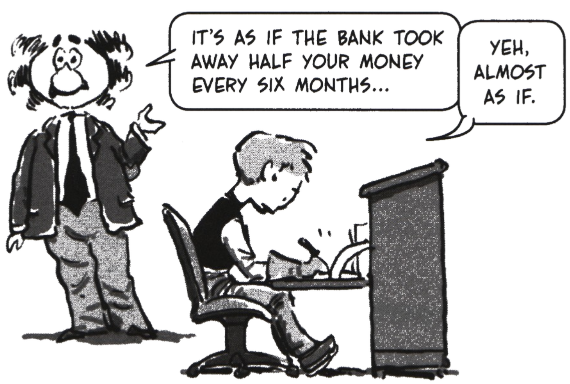


PUTTING e^{rx} IN THE DENOMINATOR CREATES A FUNCTION

$$f(x) = \frac{1}{e^{rx}} = e^{-rx}$$

THAT DIES OUT AS x GROWS LARGE, I.E., IT IS ALWAYS POSITIVE, BUT IT HEADS INEXORABLY TOWARD ZERO. THE LARGER r IS, THE FASTER THE FUNCTION DIES OUT.

e^{-rx} DESCRIBES SUCH PHENOMENA AS RADIOACTIVE DECAY, WHERE THE **DECREASE** IN RADIATION IS PROPORTIONAL TO THE AMOUNT OF RADIOACTIVE MATERIAL PRESENT, RATHER LIKE COMPOUND INTEREST IN REVERSE.

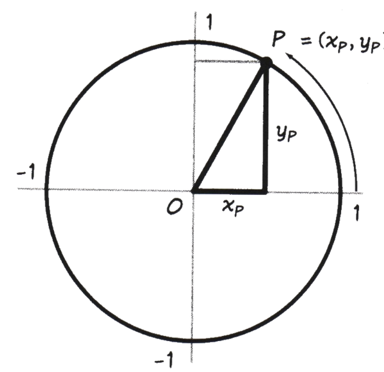


Excerpts from Gonick, Larry (2012). *The Cartoon Guide to Calculus*. New York, NY. Harper Collins.

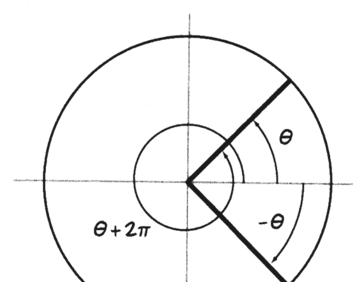
Circular Functions

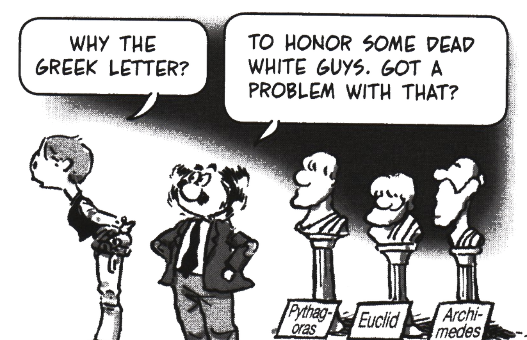
OUR FINAL ELEMENTARY FUNCTIONS ARE THE **CIRCULAR,** OR **TRIG** FUNCTIONS: THE SINE, COSINE, TANGENT, AND SECANT. THESE DESCRIBE PROCESSES THAT GO BACK AND FORTH, UP AND DOWN, IN AND OUT, LIKE TIDES AND YO-YOS.

THESE FUNCTIONS ARISE EITHER IN CIRCLES OR RIGHT TRIANGLES. HERE IS A CIRCLE OF RADIUS 1, CENTERED AT THE ORIGIN. BEGINNING ON THE x-AXIS AT (1, 0), A POINT $P = (x_P, y_P)$ ORBITS COUNTERCLOCKWISE ALONG THE RIM. YOU CAN SEE A RIGHT TRIANGLE WITH HYPOTENEUSE OP.



THE ANGLE θ (GREEK LETTER "THETA") BETWEEN OP AND THE x-AXIS IS MEASURED IN "NATURAL" UNITS, NAMELY THE **LENGTH OF THE ARC** TRAVELED BY P. THESE UNITS ARE CALLED **RADIANS.** SINCE THE CIRCLE'S CIRCUMFERENCE IS 2π, P TRAVELS 2π RADIANS IN ONE COMPLETE CIRCUIT. SMALLER ANGLES ARE PROPORTIONAL, AND MOVING CLOCKWISE GIVES NEGATIVE ANGLES. WHEN P DESCRIBES MORE THAN ONE CIRCUIT, THE ANGLE θ IS $> 2\pi$.





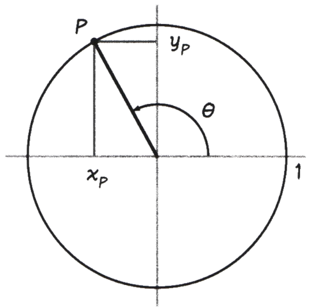
THE **SINE** AND **COSINE** OF θ ARE THE y AND x COORDINATES, RESPECTIVELY, OF THE POINT $P = (x_P, y_P)$. THE **TANGENT** OF θ IS THE RATIO y_P/x_P, WHEN $x_P \neq 0$.

$$\cos\theta = x_P$$

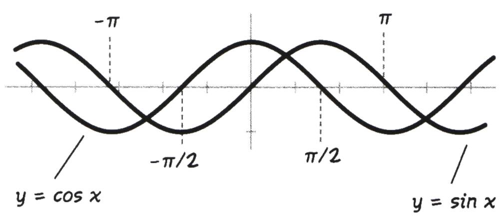
$$\sin\theta = y_P$$

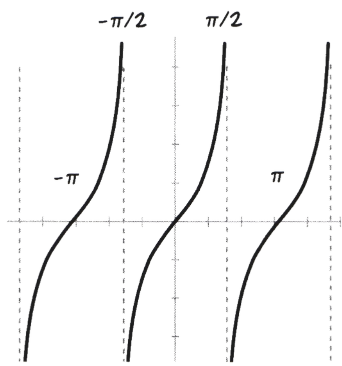
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

(YOU MAY HAVE LEARNED FROM THE ANCIENT GREEKS THAT $\sin\theta = y/r$, BUT HERE $r = 1$.)



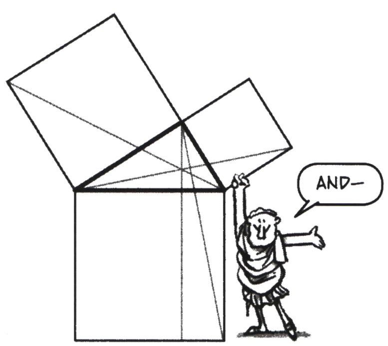
THE SINE AND COSINE OSCILLATE BETWEEN −1 AND 1, REPEATING THEMSELVES EVERY 2π RADIANS. THE TANGENT REPEATS AFTER EVERY π RADIANS. THE TANGENT ZOOMS OFF TO INFINITY AT THE ODD HALVES OF π, WHERE THE COSINE IS ZERO.





WE WILL ALSO OCCASIONALLY MENTION THE **SECANT** OF θ, WHICH IS THE RECIPROCAL OF THE COSINE, DEFINED WHEN $\cos\theta \neq 0$.

$$\sec\theta = \frac{1}{\cos\theta}$$



PYTHAGORAS GIVES US THIS HIGHLY USEFUL EQUATION

$$\sin^2\theta + \cos^2\theta = 1$$

WHICH ALSO AMOUNTS TO

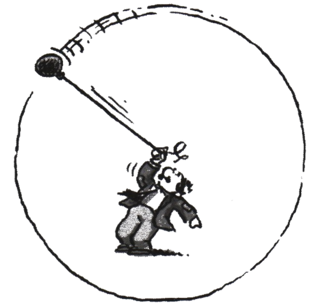
$$\sec^2\theta = \tan^2\theta + 1$$

BECAUSE

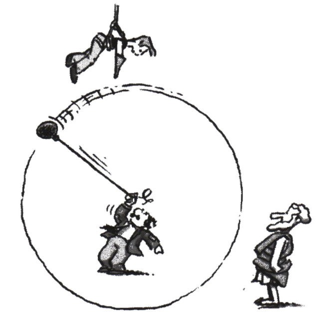
$$\sec^2\theta = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$$

Excerpts from Gonick, Larry (2012). *The Cartoon Guide to Calculus*. New York, NY. Harper Collins.

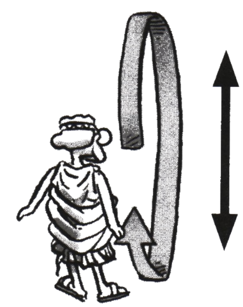
ONE WAY TO VISUALIZE THE SINE AND COSINE IS TO IMAGINE THE POINT P IS A WEIGHT BEING SPUN AROUND AT THE END OF A 1-METER ROPE.



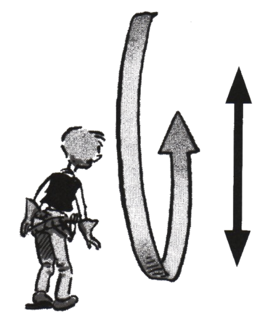
IMAGINE TWO OBSERVERS VIEWING THE CIRCLE EDGE-ON. ONE LOOKS ALONG THE x-AXIS, AND THE OTHER LOOKS DOWN THE y-AXIS.



x-GUY SEES THE WEIGHT START AT EYE LEVEL, THEN BOB UP AND DOWN, UP AND DOWN, UP AND DOWN. HE SEES THE y-VALUES, OR SINE.



y-GIRL, LOOKING DOWN, SEES **EXACTLY** THE SAME BACK-AND-FORTH MOTION, EXCEPT THAT THE WEIGHT START AT THE TOP OF ITS CYCLE. SHE SEES THE COSINE.



THIS CLEARLY SHOWS WHY THE SINE AND COSINE HAVE IDENTICAL GRAPHS, EXCEPT THAT ONE IS DISPLACED SIDEWAYS BY $\frac{\pi}{2}$.

$$\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$$

ALSO, SINCE $\cos(-\theta) = \cos\theta$,

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

AND

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

AND, AS I HOPE YOU'VE ALREADY LEARNED SOMEWHERE, THERE ARE COUNTLESS OTHER TRIGONOMETRIC IDENTITIES:

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

ETC.!