Expressions and Exponents

An *expression* is a bunch of numbers and variables joined by operations. We call a cluster of numbers and variables joined by multiplication and/or division a *term*. Two terms are considered "like" if they contain the same variables each taken to the same power. When several terms are added or subtracted in an expression, we can simplify by combining each pair of like terms into a single term. For instance:

$$3x^2 + 2y - 4x + 2xy + 7xy - y = 3x^2 - 4x + 9xy + y$$

If a number or variable is multiplied by itself many times, we call that raising a number to a power and replace the multiple copies with a single superscripted (ie. smaller and raised) number called an *exponent*. For example:

$$x \cdot x \cdot x \cdot x \cdot x = x^5$$

The 5 on the right side of the equation is an exponent, and we'd say that x is raised to the fifth power. Manipulation of exponents involves a few properties:

$x^0 = 1$	Any quantity raised to the zero power is equal to 1.
$x^a x^b = x^{a+b}$	The multiplication of the same quantity raised to various powers is the quantity raised to the sum of the exponents.
$\frac{x^a}{x^b} = x^{a-b}$	The division of the same quantity raised to various powers is the quantity raised to the difference of the exponents.
$x^{-a} = \frac{1}{x^a}$	A quantity raised to a negative power is the reciprocal of the quantity raised to the inverse of the original exponent.
$(x^a)^b = x^{ab}$	A quantity with an exponent raised to another power is the quantity raised to the product of the exponents.
$\left(\frac{x^a y^b}{z^c}\right)^d = \frac{x^{ad} y^{bd}}{z^{cd}}$	When raising an entire term to a power, each number or variable in the term must be raised to that power.
$x^{\frac{a}{b}} = \sqrt[b]{x^a}$	A quantity raised to a fractional power is the same as the denominator root of the quantity raised to the power of the numerator.

When we multiply two expressions by each other, according to the distributive property we need to multiply each term from the first expression by each in the second. In particular:

$$(x+a)(x+b) = x^2 + ax + bx + ab$$

Going in the inverse direction we can use the *quadratic formula*, which says if $ax^2 + bx + c = 0$ then:

$$x = \frac{-b - \sqrt{(b^2 - 4ac)}}{2a}$$