

PreAlgebra review

PreAlgebra

Multiplication Facts

$$0 \times \text{number} = 0$$

$$1 \times \text{number} = \text{number}$$

$$2 \times \text{number} = \text{an even number}$$

$$3 \times \text{number} = 3 \text{ divides sum of digits}$$

$$4 \times \text{number} = 4 \text{ divides last two digits}$$

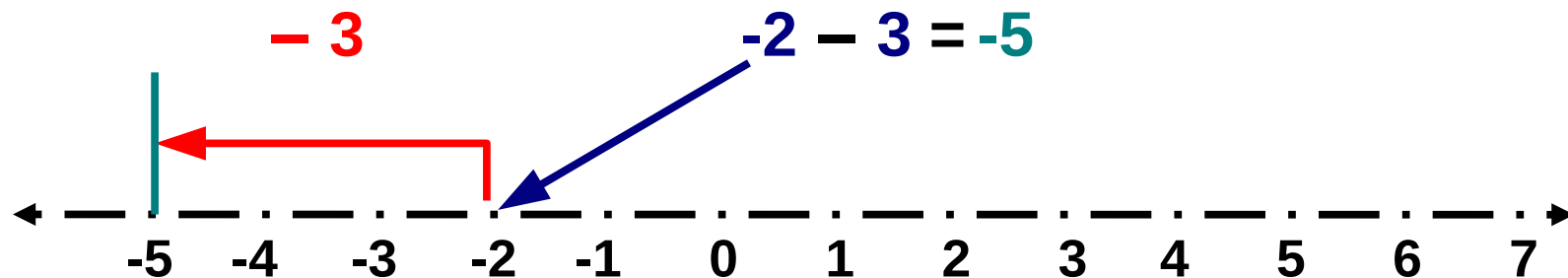
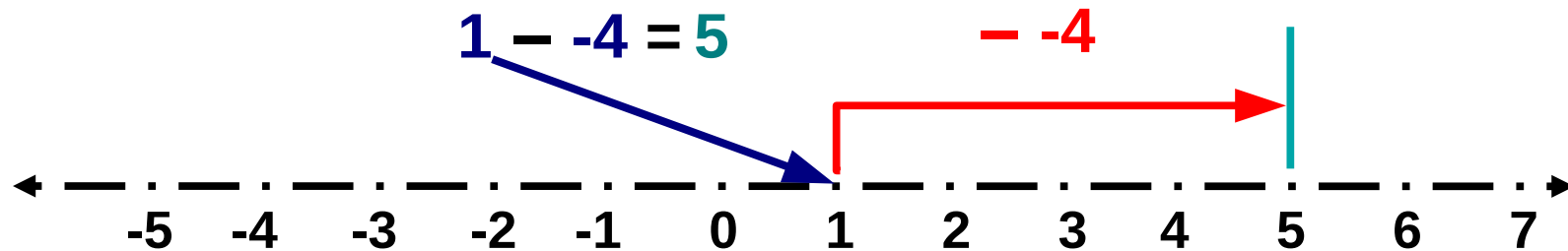
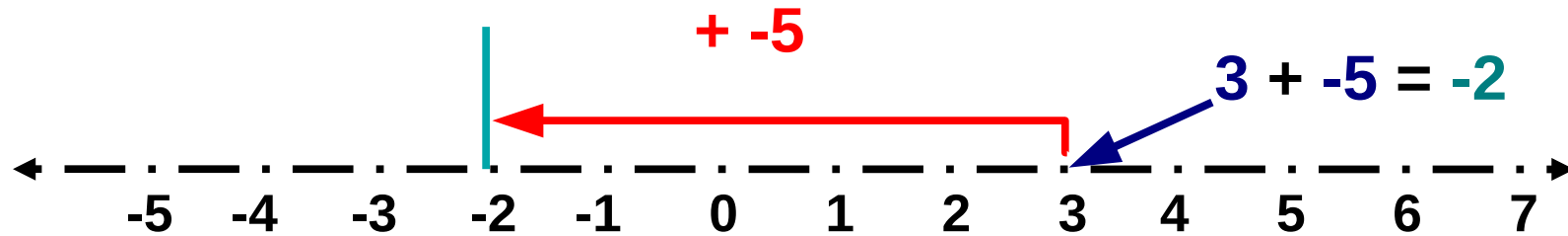
$$5 \times \text{number} = \text{number ending in } 0 \text{ or } 5$$

$$6 \times \text{number} = \text{divided by } 2 \text{ and } 3$$

$$9 \times \text{number} = 9 \text{ divides sum of digits}$$

+	x	+	is	+
+	x	-	is	-
-	x	+	is	-
-	x	-	is	+

Integer Addition and Subtraction



Squares and Square Roots

Addition

$$3 + 3$$

$$5 + 5 + 5 + 5 + 5$$

Multiplication

$$= 3 \cdot 2$$

$$= 5 \cdot 5$$

$$5 \cdot 5 \cdot 5 \cdot 5 = 5^4$$

$$= 5^2$$

Exponents

	--- 3 ---		
3	1	2	3
	4	5	6
	7	8	9

Square of 3

$$= 3 \cdot 3 = 3^2 = 9$$

3^2 is NOT $3 \cdot 2$

Square **Root** of 9

$$= \sqrt{9} = 3 \text{ NOT } 81$$

repeated division by the same number

$$\sqrt{400} = \sqrt{4 \times 100} = 2 \times 10 = 20$$

$$\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$$

$$\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = 4$$

$$\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$$

PreAlgebra

Order of Operations

Parenthesis

$$5 (3 + 2)$$

Exponents

$$6 + 3^2$$

Multiplication

Division

$$3 + 6 / 2 - 5 \cdot 4$$

Addition

$$= 3 + 3 - 20$$

Subtraction

$$= -14$$

Please Excuse My Dear Aunt Sally

$$5 - 9 / 3 + 2 \cdot 4^2 - 6 (8 - 5)$$

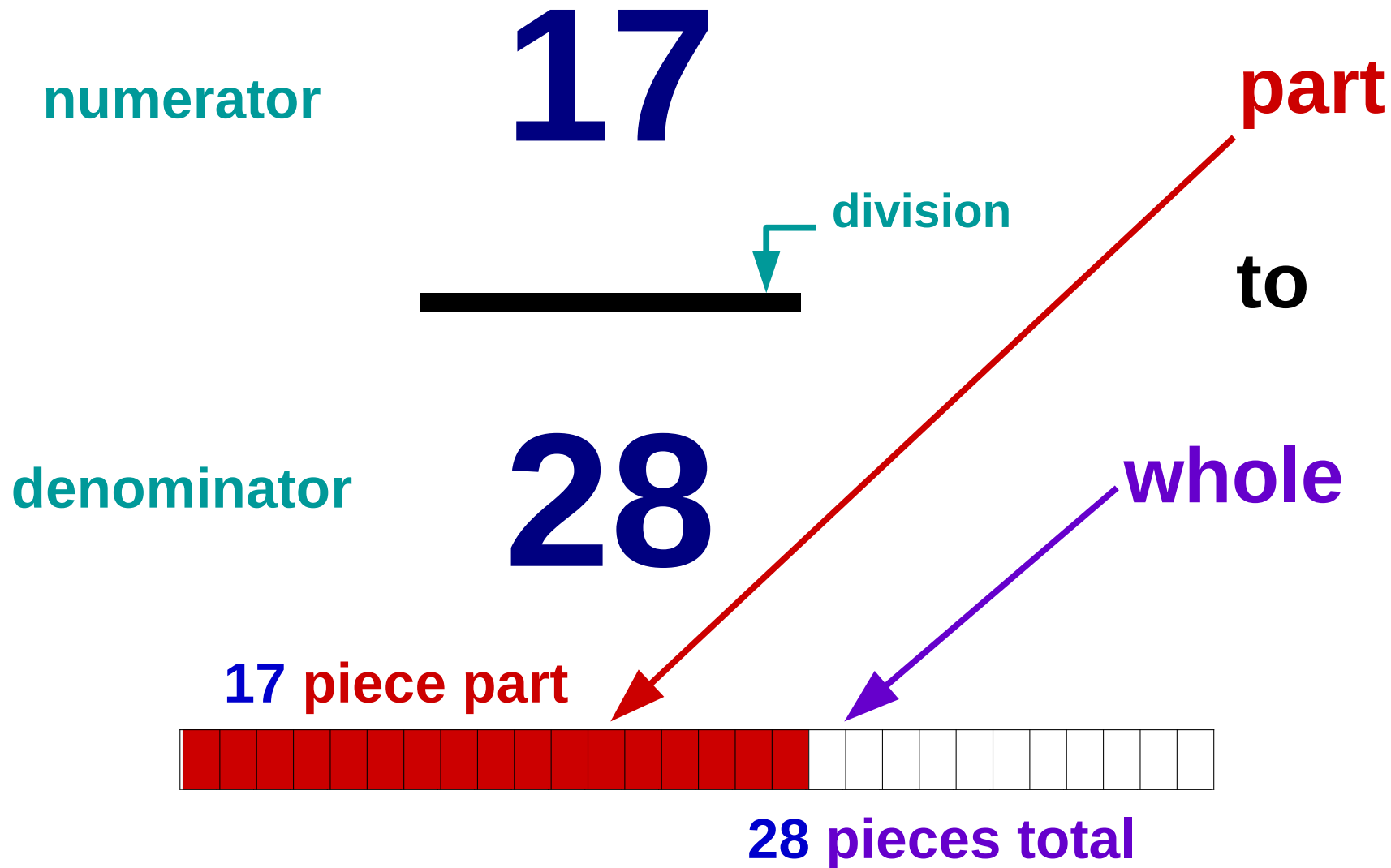
$$= 5 - 9 / 3 + 2 \cdot 4^2 - 6 \cdot 3$$

$$= 5 - 9 / 3 + 2 \cdot 16 - 6 \cdot 3$$

$$= 5 - 3 + 32 - 18 = 16$$

Fraction

two **numbers** in comparison



Comparing Fractions

$$\frac{1}{2} \cdot \frac{2}{2} = \frac{2}{4} \cdot \frac{2}{2} = \frac{4}{8} \cdot \frac{7}{7} = \frac{28}{56} \qquad \frac{2}{3} \cdot \frac{4}{4} \stackrel{?}{=} \frac{8}{12}$$

= equal to
> greater than
< less than

$$\frac{18}{30} \cdot \frac{6}{6} \cdot \frac{3}{5} \stackrel{?}{=}$$

$$\frac{22}{48} \div \frac{2}{2} = \frac{11}{24} \stackrel{?}{=} \frac{33}{72}$$

$$\frac{10}{12} \cdot \frac{2}{2} \cdot \frac{5}{6} \stackrel{=}{=} \frac{10}{12}$$

$$\frac{39}{45} = \frac{3 \cdot 13}{3 \cdot 15} > \frac{38}{45}$$

Reducing Fractions

$$\frac{1}{2} \cdot \frac{2}{2} = \frac{2}{4} \cdot \frac{2}{2} = \frac{4}{8} \cdot \frac{7}{7} = \frac{28}{56} \dots$$

**infinitely
many**

$$\frac{40}{65} \div \frac{5}{5} = \frac{8}{13}$$

**lowest
terms**

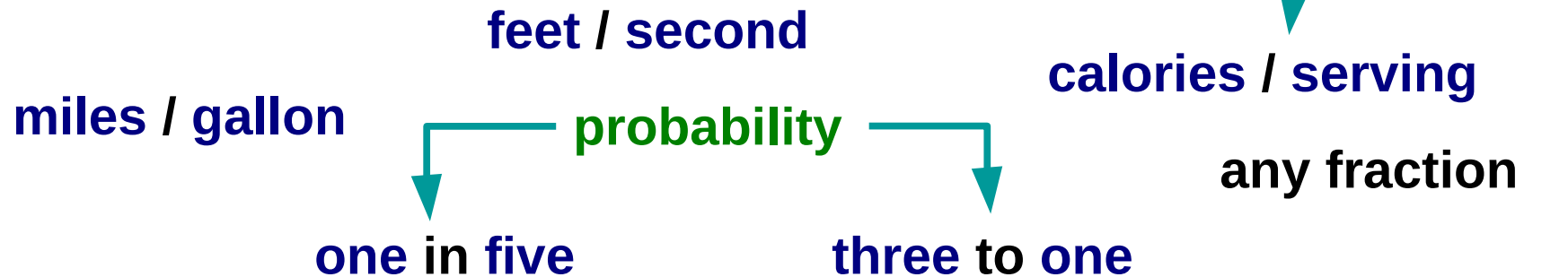
if there is a whole **number** that can divide into both the numerator and denominator of the fraction without remainder, the fraction can be reduced

$$\frac{18}{30} \div \frac{2}{2} = \frac{9}{15} \div \frac{3}{3} = \frac{3}{5}$$

$$\frac{6}{6} = \frac{18}{30}$$

Ratio

a **ratio** is a comparison of two **quantities**



x / y (purple) ← **variables** (green) studied through calculus

change (red) **Δx / Δy** (red/purple)

Percent

a **ratio** of one **quantity** to 100 per **100**

$$x \div 100 \qquad \frac{x}{100} \qquad x \%$$

Ratio Conversion

fractions

$$\frac{14}{100} = \frac{7}{50}$$

$$\frac{7}{100}$$

$$\frac{32}{1000} = \frac{4}{125}$$

$$\frac{179}{100}$$

division

$$\frac{3}{4}$$

decimals

.14

.07

.032

1.79

.75

percents

14%

07%

3.2%

179%

75%



Ratio Conversion

fractions

$$\frac{2}{100} = \frac{1}{50}$$

$$2\frac{57}{100}$$

$$-\frac{7}{8}$$

decimals

.02

2.57

-0.875

percents

2%

257%

-87.5%

x 100

÷ 100

Ratio Comparison

decimal to decimal

$$.20 > .14$$

= equal to
> greater than
< less than

division

fraction to fraction

$$0.6250 = \frac{5}{8} < \frac{9}{13} \approx 0.6923$$

convert each **ratio** to decimal form
with the same number of digits after the decimal point

Percent from Numbers

9 is what percentage of 36?

$$\frac{\text{is } 9}{\text{of } 36} = .25 = 25\%$$

Number from Percent

40% of what number is 32?

$\frac{40}{100} = \frac{32}{x}$ **is of** **cross-multiply**

$$\frac{100 \cdot 32}{40} = \frac{40 \cdot x}{40}$$

$$\frac{100 \cdot 32}{40} = 80 = x$$

Percent of Numbers

What is **35** percent of **250**?

$$\begin{array}{l} \text{is} \\ \hline \text{of} \end{array} \quad \frac{35}{100} \quad \begin{array}{c} \text{X} \\ \text{X} \end{array} \quad \frac{x}{250} \quad \quad x = \frac{35 \cdot 250}{100} = 87.5$$

$$35\% = 0.35$$

of means **multiply** for
fractions, decimals,
and percents

$$0.35 \cdot 250 = 87.5$$

Uses of Percentages

as ratios to a reference value

36 students in a class of **400**

9%

for changes in a reference value

price increase from \$**56** to \$**77**

+37.5%

for comparisons between different values

comparing **9"** to **12"**

33.3% more

compared value – reference value

reference value

x 100%

Percent Change

from **16** to **20**

positive value
means **increase**

$$\frac{20 - 16}{16} = \frac{4}{16} = 0.25 = 25\%$$

from **7.2** to **6.0**

negative value
means **decrease**

$$\frac{6.0 - 7.2}{7.2} = \frac{-1.2}{7.2} = -0.\overline{16} \approx -16.7\%$$

new value – original value

original value

x 100

Change by Percent

30% increase from 70

positive value
means **increase**

$$70 + (70 \times 30 \div 100) = 70 + 21 = 91$$

15% decrease from 120

negative value
means **decrease**

$$120 - (120 \times 15 \div 100) = 120 - 18 = 102$$

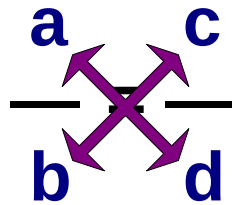
original value $\pm$ (original value $\times$ percent $\div$ 100)

depends



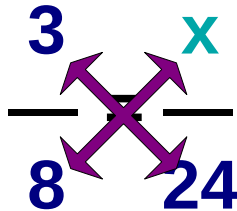
Proportion

a statement that two **ratios** are equal is a **proportion**



$$a \cdot d = b \cdot c$$

cross-multiply



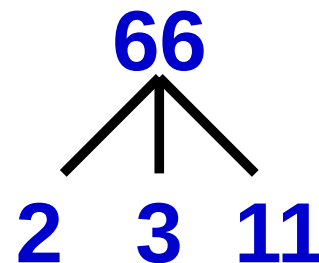
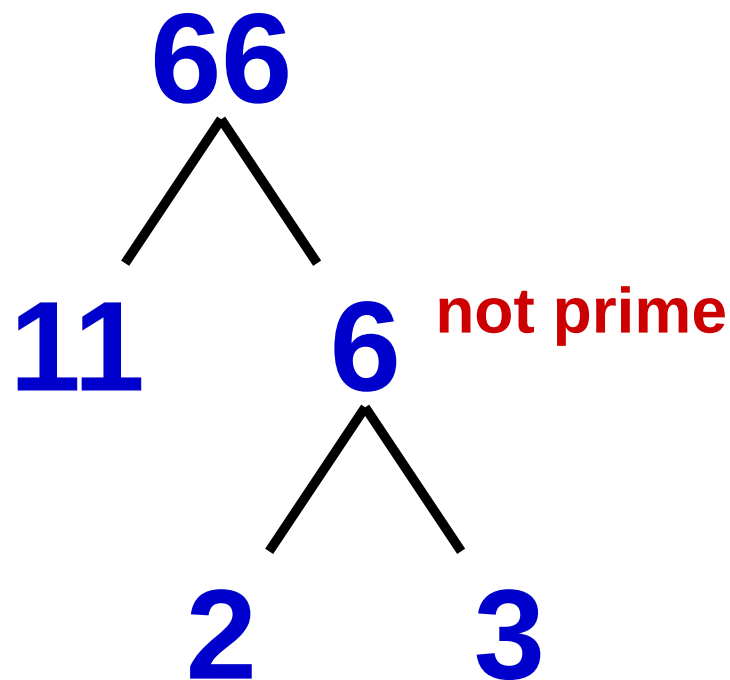
$$\frac{3 \cdot 24}{8} = \frac{8 \cdot x}{8}$$

$$\frac{3 \cdot 24}{8} = x$$

$$\frac{5}{7} = \frac{10}{x}$$

$$x = \frac{7 \cdot 10}{5} = \frac{70}{5} = 14$$

Factoring



prime numbers

2
3
5
7
11
13
17
...

$$2 \cdot 3 \cdot 11 = 66$$

breaking a number down into a list of all the **prime** numbers that divide the original number without remainder

Multiplying and Dividing Fractions

$$\begin{array}{r} 3 \\ \hline 4 \end{array} \times \begin{array}{r} 1 \\ \hline 2 \end{array} = \begin{array}{r} 3 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 5 \\ \hline 6 \end{array} \div \begin{array}{r} 1 \\ \hline 3 \end{array} = \begin{array}{r} 5 \\ \hline 6 \end{array} \times \begin{array}{r} 3 \\ \hline 1 \end{array} = \begin{array}{r} 15 \\ \hline 6 \end{array} \div 3 = \begin{array}{r} 5 \\ \hline 2 \end{array}$$

• whole numbers $9 = \frac{9}{1}$

• mixed numbers

$$2\frac{+4}{\times 5} = \frac{14}{5} \text{ improper fraction}$$

• reduction $\begin{array}{r} 1 \\ \cancel{4} \\ \hline 7 \end{array} \times \begin{array}{r} 3 \\ \hline \cancel{8} \\ 2 \end{array} = \begin{array}{r} 3 \\ \hline 14 \end{array}$

+	x	+	is	+
+	x	-	is	-
-	x	+	is	-
-	x	-	is	+

Addition and Subtraction of Fractions

$$4\frac{1}{3} - 1\frac{7}{9} =$$

1) convert mixed to improper

$$4\frac{1}{3} = \frac{3 \cdot 4 + 1}{3} \qquad 1\frac{7}{9} = \frac{9 \cdot 1 + 7}{9}$$

2) change fractions to common **denominator**

$$\frac{13}{3} \cdot \frac{3}{3} = \frac{39}{9} \qquad \frac{16}{9}$$



3) add or subtract **numerators** only

$$\frac{39}{9} - \frac{16}{9} = \frac{23}{9} = 2\frac{5}{9}$$

division

Measures of Central Tendency

data set: { 5, 8, 5, 6, 4, 1, 6 }

range largest minus smallest value

$$8 - 1 = \underline{7}$$

mode most common value in a set

5, 8, 5, 6, 4, 1, 6

median middle value in an ordered set

1, 4, 5, 5, 6, 6, 8, 9


$$\frac{5 + 6}{2}$$

mean sum divided by number of values

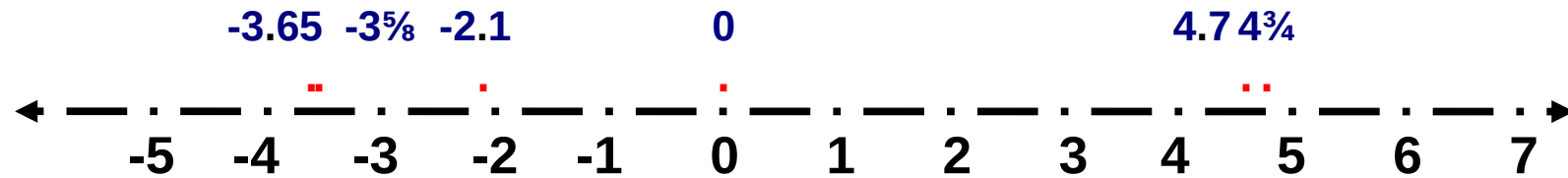
$$1 + 4 + 5 + 5 + 6 + 6 + 8 = 35$$

$$\frac{35}{7} = \underline{5}$$

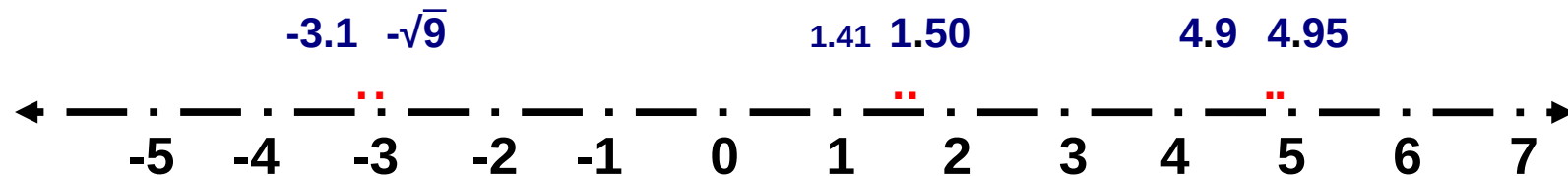
average

Ordering of Rational Numbers

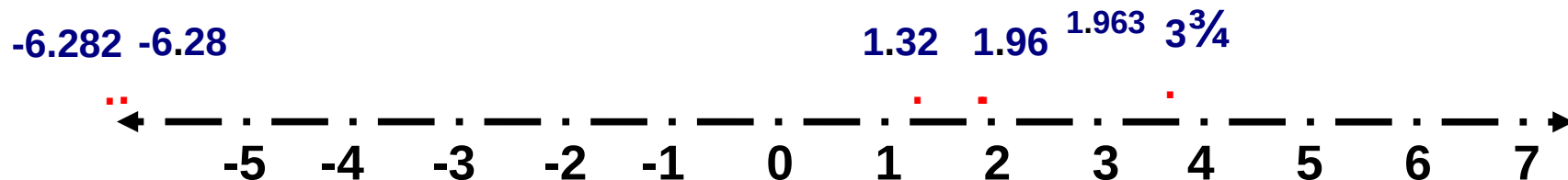
$$S_1 = \{ 0, -2.1, 4\frac{3}{4}, -3.65, 4.7, -3\frac{5}{8} \}$$



$$S_2 = \{ 4.95, -\sqrt{9}, 4.9, -3.1, 1.41, 1.5 \}$$



$$S_3 = \{ 1.32, -6.282, 1.96, -6.28, 3\frac{3}{4}, 1.963 \}$$



Powers of Ten

dividing by powers of ten
involves moving
the decimal point to the left

tenths place
hundredths place
thousandths place

1492.357

ones place
tens place
hundreds place
thousands place

$$1492.357 \div 1,000$$

$$= 1492.357 \times \frac{1}{1000}$$

$$= 1.492357$$

$$1492.357 \times 10,000$$

$$= 14,923,570$$

multiplying by powers of ten
involves moving
the decimal point to the right

Precision and Rounding

tenths place
 hundredths place
 thousandths place
1492.357
 ones place
 tens place
 hundreds place
 thousands place

to tenths

1492.4

within
0.05

to nearest whole number

1492

within
0.5

$$2 \times 10^0 = 2 \times 1 \quad 2$$

$$9 \times 10^1 = 9 \times 10 \quad 90$$

$$4 \times 10^2 = 4 \times 100 \quad 400$$

$$1 \times 10^3 = 1 \times 1000 \quad 1000$$

$$3 \times 10^{-1} = 3 \times 1/10 \quad 0.3$$

$$5 \times 10^{-2} = 5 \times 1/100 \quad 0.05$$

$$7 \times 10^{-3} = 7 \times 1/1000 \quad + \underline{0.007}$$

precision to hundredths

1492.36

within
0.005

Rounding

hundred **hundredth**



867.5309

for the digit after
0-4 round down
5-9 round up

to the nearest hundredth

867.53

to the nearest hundred

900

to the nearest ten-thousandth

867.5309